Physical observables have been the central theme in my research. The S-matrix, form factors and correlation functions are just some examples of physical observables to which it is the theory’s (or theorist’s) job to give a definite prediction. Take example the S-matrix, which describes the probability for certain incoming states to interact and transform into particular outgoing states. In quantum field theory, we generally compute the S-matrix by summing over a series of Feynman diagrams:

On the other hand, if your favorite theory is string theory, then you would sum over different two-dimensional topologies with insertions,

But no matter what your theory is, at the end of the day, it needs to give a sensible physical observable. Here by sensible we mean that the S-matrix must satisfy unitarity, locality and preserve causality. These are universal properties of the S-matrix, and in general are hard to satisfy. A famous example is the scattering of four massive vectors, violates unitarity at large transverse momenta. It is only after the inclusion of the Higgs boson does the high energy behavior becomes tamable. Similarly graviton S-matrix also violates unitarity at high energy whose violation are intimately tied to the production of black holes. From the above examples, we see that by studying the properties of the S-matrix, the low energy or low order in perturbation results often contain tantalizing information on the correct “path” to an ultra-violet completion.

On the other hand, we can turn the argument on its head. Instead of trying to construct a theory first and then labor to extract its predictions, we start with the physical observable itself, using symmetry properties to determine its lowest order constituent, then apply the constraint of unitarity and locality to build the entire interacting theory order by order in perturbation theory:

Indeed if we restrict ourselves to massless particles, the three-point S-matrix for massless vectors and tensors is sufficient for us to reconstruct Yang-Mills theory and General relativity. Note that there was no need to introduce the notion of gauge symmetry nor differential geometry! Thus by putting the physical observable at the center of our attention, we can free ourselves from the confine of formalisms that may have been helpful in the past to construct theories, but may no longer be relevant for the problem at hand.

My current research can be separated into the following broad questions.

1. WHAT EXACTLY IS WRONG WITH QUANTUM GRAVITY AND IS STRING THEORY THE UNIQUE SOLUTION?

When we say that general relativity and quantum mechanics are incompatible, what we really mean is that when we apply the methods of quantum field theory to the classical Einstein-Hilbert action and try to calculate the S-matrix, we get an infinite number of infinities. You may wonder how do we know that, since surely no one has actually done the infinite number of calculations before. Indeed no one has. The argument goes as follows: (1) The Einstein-Hilbert action is invariant under diffeomorphism, and thus any quantity that is calculated

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1 Graph taken from Bin Gong et. al. Phys. Rev. D83 (2011) 114021
using this action must also be diffeomorphism invariant.

(2) The coupling constant is dimensionful, which means that each order in perturbation expansion will generate terms of different mass dimensions. (3) If there is an infinity, then from the view point of renormalization it must be associated with a local operator that has the right dimension and diffeomorphism invariant. (4) Since one can show that there are infinite number of diffeomorphism invariant operators in gravity, then according to Gell-Mann’s Totalitarian Principle “everything not forbidden is compulsory”, there must be an infinite number of infinities. What string theory does is that it introduces an infinite number of massive particle states with a particular coupling such that the infinities can be canceled out.

However, as you have noticed in the above description, diffeomorphism invariant has been the linchpin of the entire argument. If gravity has more symmetries or structures than diffeomorphism (or its supersymmetrized version), then the whole argument falls apart. Why do we believe there are more structures? Because explicit computations have revealed examples where although symmetry satisfying counter term exists, there were no infinities! For example, using modern computation techniques such as unitarity methods [1], we’ve explicitly computed the three-loop S-matrix for $N = 4$ super gravity [2], where it is known that a symmetry satisfying counter term $R^4$ exists. The answer is given by the following diagrams:

![Diagrams](image)

Each of the diagram contains an infinity as expected, and they are listed as follows:

<table>
<thead>
<tr>
<th>Graph</th>
<th>$(\text{divergence})/(\text{(12)}^2 \cdot \text{34})^2 \text{stA} \text{tree}(\text{23})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)-(d)</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>$263 \cdot 1 + 205 \cdot 1 + 276 \cdot 1 + 217 \cdot 1 + 214 \cdot 1 + 263 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(f)</td>
<td>$-175 \cdot 1 - 11 \cdot 1 + 329 \cdot 1 + 290 \cdot 1 + 329 \cdot 1 + 290 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(g)</td>
<td>$38 \cdot 1 - 41 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(h)</td>
<td>$-3 \cdot 1 - 41 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(i)</td>
<td>$12 \cdot 1 + 102 \cdot 1 + 102 \cdot 1 + 102 \cdot 1 + 102 \cdot 1 + 102 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(j)</td>
<td>$-38 \cdot 1 - 41 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(k)</td>
<td>$5 \cdot 1 + 89 \cdot 1 + 1182 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 214 \cdot 1 + 105987 \cdot 1$</td>
</tr>
<tr>
<td>(l)</td>
<td>$263 \cdot 1 + 205 \cdot 1 + 276 \cdot 1 + 217 \cdot 1 + 214 \cdot 1 + 263 \cdot 1 + 105987 \cdot 1$</td>
</tr>
</tbody>
</table>

where $\epsilon$ is the dimensional regularization parameter indicating the divergence. Remarkably, they all add up to zero!

So as you can see, there is no divergence at all even though our previous argument said there would be. Being a physicist we would like to associate this with some hidden structure that was previously unknown. Such a structure was proposed by Bern, Carassco and Johansson (BCJ) [3], as a duality between the color structure of gauge theories and its kinematics. In a nutshell it states that a general gauge theory S-matrix can be represented as:

$$A_n = \sum_{i \in \text{dia}} n_i c_i \prod_{\alpha_i} P_{\alpha_i}^2$$

where $i$ labels all cubic diagrams, $\alpha_i$ labels the propagators ($P_{\alpha_i}^2$) in each diagram, $n_i$ are the kinematic numerators and $c_i$ are the color factors (the global part of the gauge group). The duality states that one can find a representation such that the algebraic relations between the $c_i$s of different diagram is identical to that of the $n_i$s. Furthermore, we proved that once such a duality is established [4], the gravity S-matrix is simply

$$M_n = \sum_{i \in \text{dia}} n_i n_i \prod_{\alpha_i} P_{\alpha_i}^2.$$  

In other words, the interaction of degrees of freedom we associate with the fluctuation of space-time, is given by the product of interaction of a normal gauge theory!

This new duality has been shown to be directly responsible to the absence of infinities in certain cases [5], even though symmetry satisfying counter term exists. Furthermore in three-dimensions, this duality becomes enhanced where we have a triality between the usual Lie-2 algebra based Yang-Mills theory, a Lie-3 algebra based Chern-Simons matter theory and gravity [6]:

![Diagram](image)

Thus given all these new found structure of the S-matrix for quantum gravity, it is ripe to challenge whether string theory is the unique finite gravity theory. Might gravity allow more than one possible ultra-violet completion?

2. HIDDEN SYMMETRIES AND NEW STRING THEORIES

Why do we believe there must be hidden symmetries? Simple, consider the calculation of a five-point S-matrix. In QFT, the answer is the following:

\[ \langle 13 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \]  

where \( \langle ij \rangle \equiv \lambda^a_i \lambda^a_j \) and \( \lambda_i \) is related to the momentum via \( (\lambda^a_i)^* \lambda^a_i = p^a_{\mu} \). The simplicity of the final answer begs for a better description of how particles interact.

Recently there has been an intriguing new way of thinking what the S-matrix is, for which the simplicity of the final answer is manifest. Instead of thinking of the \( n \)-point S-matrix as a function of kinematic invariants, the same information can be recast into the planes in an \( n \)-dimensional space. The S-matrix is then associated with special configurations of this space. For example, the fact that momentum conservation must be satisfied is reflected in that the planes that represent the amplitude must be orthogonal to the \( 2 \times n \)-dimensional plane that is \( \lambda^a_i \):

The moduli-space of planes are called a Grassmannian manifold, and Nima Arkani-Hamed et. al. shown that the S-matrix of the maximally supersymmetric Yang-Mills theory is built from different configurations of such plane [1]. Since tree-level S-matrix of maximally supersymmetric Yang-Mills contains that of reduced or non-supersymmetric ones, this covers all possible interactions of pure gauge theories. The precise configuration can be iteratively built from the fundamental building block, the three-point S-matrix plane [2]. For example one of the planes can be pictorially represented as

In the above, each white and black vertex represent the two possible three-point S-matrix plane, which is completely fixed by Lorentz symmetry.

Note that this is not unique to four-dimensions. Along with my collaborators we’ve shown that the same is true for three-dimensional Chern-Simons matter theory [3], where now instead we have fundamental four-point S-matrix plane.

Note that in this description locality, and unitarity, becomes an emergent property and no longer fundamental!

The fact that the S-matrix can be formulated in this fashion manifests a hidden symmetry: dual superconformal symmetry [4]. This symmetry is non-local in nature and hence is completely obscure from the action.
point of view and only visible by directly studying the S-matrix. The symmetry was first discovered for maximal Super Yang-Mills theory and latter in Chern-Simons matter theories [5,6]. They imply an integrable structure in the S-matrix that is currently under intense investigation.

Interestingly these special Grassmannian configurations, when combined into the S-matrix, can be rewritten as a map that converts a Riemannian manifold with punctures to a Grassmannian. This is very much like a string theory, and indeed it is! We usually associate string theory as the theory of strings propagating in space-time. However we can also consider strings propagating in momentum space, or its on-shell version: twistor space. Witten proposed a new twistor-string theory which is topological in nature and whose string scattering amplitudes, which is non-trivial due to instanton effects, gives the S-matrix of maximal Super Yang-Mills theory. This is precisely the underlying picture that yields the map between Riemannian manifold and the Grassmannian. With my collaborator [7], we’ve also found a similar map for the three-dimensional Chern-Simons matter theory, thus hinting at another new string theory yet to be discovered. Indeed not long afterwards, one was found [8]!


3. HOW TO THINK ABOUT QUANTUM FIELD THEORIES WHEN THERE IS NO LAGRANGIAN?

From the previous discussion, it should be apparent that often times Lagrangians are not the best starting point to define a quantum field theory. In fact, it is believed that there are a large class of strongly coupled systems that do not have a Lagrangian formulation. Therefore it is interesting to reconsider aspects of physics that are traditionally discussed in the context of Lagrangians and path integrals. For example the notion of renormalizability and gauge anomalies.

As discussed previously, renormalizability are usually associated with the combined analysis of the symmetries of the Lagrangian and the dimension of the coupling constant. In four-dimensional theories with massless particles, the one-loop S-matrix can be expressed in terms of a basis of scalar integrals and rational terms. Since the scalar bubble integrals are the only UV divergent integrals, the sum of the bubble coefficients equals the tree-level amplitude times a proportionality constant that is related to the one-loop beta function coefficient $\beta_0$. By studying how the S-matrix satisfies this criteria, we revealed new structures within the one-loop S-matrix that insures the renormalization conditions are met.

Traditionally, gauge anomalies are associated with the fact that available regulators break the symmetry of the classical action. While this is certainly true, it is rather artificial on two counts: (1) gauge symmetry is not a real symmetry, and thus if there is inconsistency with the theory, one must be able to formulate the inconsistency in terms of violations of physical properties from the outset. (2), to blame the regulator as the culprit is rather artificial. To put it bluntly, how do we know it is really just a matter of lack of imagination rather than serious sickness with the theory?

This problem becomes sharper in the modern on-shell approach, where the perturbative S-matrix is constructed iteratively using on-shell building blocks with manifest unitarity. As only gauge invariant quantities enter in the intermediate steps, the notion of gauge anomaly is meaningless. In [2] we demonstrate that while the unitarity-methods automatically lead to a unitary S-matrix, the rational terms that are required to enforce locality, invariably give rise to inconsistent factorization channels in chiral theories. In four-dimensions, the absence of such inconsistencies implies the vanishing of the cubic Casimir of the gauge group. In six-dimensions, if the symmetric trace of four generators does not vanish, the rational term develops a factorization channel revealing a new particle in the spectrum: the two-form of the Green-Schwarz mechanism. Thus in the purely on-shell construction, the notion of gauge-anomaly is replaced by the irreparable tension between the requirement of locality and the unitary for the S-matrix. As the S-matrix is finite, this tension is independent of regulators.