Classical Electrodynamics (I) PhD Qualifying Exam (5 problems)

Note: 1. This is a closed-book exam.
2. Terms and notations follow Jackson.

1. Answer "yes" or "no" to the following statements (no explanation is required). (45%, 3 points for each correct answer; –1 point for each wrong answer; 0 point for each unanswered question. If total points <0, it will be regarded as 0.)
   (1) In electrostatics, the potential ϕ is continuous and electric field **E** is discontinuous across a single layer of surface charge distribution.
   (2) In electrostatics, ϕ is discontinuous and **E** is continuous across a dipole layer.
   (3) The static electric field **E** can be calculated by using $\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$ alone.
   (4) The integral form of Gauss's law for **E** is *mathematically* equivalent to the differential form of Gauss's law for **E** whether **E** is static or time-dependent.
   (5) If $\oint_S \mathbf{A}(\mathbf{x}) \cdot \mathbf{n} da = 0$ for any *closed* surface S (da is a differential area of the surface and $\mathbf{n}$ is its outward normal), it implies $\mathbf{A}(\mathbf{x}) = 0$ everywhere.
   (6) The electric field E on the surface of a conductor with static surface charge density σ is $E=\sigma/\varepsilon_0$ even if the surface is curved or σ is non-uniform.
   (7) $\varepsilon_0 E^2(\mathbf{x})$ and $q\delta(\mathbf{x}-\mathbf{x}_0)\Phi(\mathbf{x})$ have the same dimension
   (8) Electric polarization **P** and electric dipole moment **p** do not have the same dimension.
   (9) The polarization charge density ($\rho_{pol} = -\nabla \cdot \mathbf{P}$) is only a convenient definition. It does not represent real charges.
   (10) The reason a 2.45 GHz microwave oven can heat food is because water molecules in food have a resonant frequency at approximately 2.45 GHz.
   (11) The dipole moment **p** of a distribution of electrical charges is always independent of its point of reference.
   (12) The magnetic field energy density, $w_b = \mathbf{H} \cdot \mathbf{B}$, is derived for a *linear* medium, i.e. $\mathbf{B} = \mu \mathbf{H}$ with μ independent of **B**.
   (13) The magnetization current density ($\mathbf{J}_M = \nabla \times \mathbf{M}$) is only a convenient definition. It does not represent real current.
   (14) A static magnetic field can penetrate through a good conductor with $\mu = \mu_0$ as if there were no conductor present.
   (15) Newton's law states that the total mechanical momentum of an isolated system is conserved. This is also true in electrodynamics.

2. (a) Write down the 4 microscopic Maxwell equations in differential form. (4%)
   (b) Show that conservation of charge is implicit in this set of equations. (5%)
   (c) Show that the electric field **E** and magnetic induction **B** can be represented by a scalar potential ϕ and a vector potential **A**. (6%)

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3. A plane electromagnetic wave with instantaneous fields $\mathbf{E}_i$ and $\mathbf{B}_i$ is incident normally from the free space onto a stationary and perfectly-conducting plane. The reflected wave has instantaneous fields $\mathbf{E}_r$ and $\mathbf{B}_r$.

   (a) What is the relation between $\mathbf{E}_r$ and $\mathbf{E}_i$ on the conductor? Give your reason. (8%)

   (b) What is the relation between $\mathbf{B}_r$ and $\mathbf{B}_i$ on the conductor? Give your reason. (8%)

4. Verify Poynting's theorem for the case of a long, straight conducting wire of radius a and conductivity σ, which carries a direct current I. [The resistance per unit length of the wire is $R = 1/(\sigma\pi a^2)$]. (16%)

5. A measuring device is disturbed by the following influences. How would you separately protect the device from each of the following influences?

   (a) High-frequency electromagnetic fields. (4%)

   (b) A static magnetic field. (4%)