古典電力學(1)博士班資格考(共 6 題)

Note: 1. Equations in the problems are in SI unit system.
2. Notations follow Jackson.
3. This is a closed-book exam.

1. (a) The electron has a charge of $e = 1.6\times10^{-19}$ C. Assume it has a spherical shape with a radius of 3×10^{-15} m and its charge is uniformly distributed in its volume. Calculate the electric field on the surface of the electron. (5%)

   (b) If the result in (a) is the peak electric field of a monochromatic plane electromagnetic wave in free space. What is the peak magnetic induction (B) of this wave? What is the peak power per unit area of this wave? (10%)

   Useful information:

   $\oint_S \mathbf{E}\cdot\mathbf{n}\, da = \frac{1}{\varepsilon_0}\int_V \rho(\mathbf{x})d^3x$ [Gauss law]; $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ [Faraday's law];

   $\mathbf{S} = \mathbf{E}\times\mathbf{H}$ [Poynting vector]; $\varepsilon_0 = 8.85\times10^{-12}$ F/m; $\mu_0 = 1.26\times10^{-6}$ H/m.

2. Show that, inside a linear and uniform dielectric medium of permittivity ε, the polarization charge density (ρ_p) is related to the net charge density (ρ_{net}) by

$$\rho_p = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}\rho_{net}$$
,

   where ε_0 is the permittivity of free space. (15%)

3. Two identical, perfectly conducting loops are far apart and share the same axis. Each has self-inductance L and a current I flowing in the same direction, so the energy in each loop is $LI^2/2$. They are then brought together and superposed.
   (a) What is the final current in each loop? (10%)
   (b) What is work done in bringing the two loops together? Is the work done on the loops or by the loops? (10%)

4. Consider a plane wave propagating in a dielectric medium characterized by a complex electric permittivity of $\varepsilon = \varepsilon_0(1 + 0.001i)$ and a complex magnetic permeability of $\mu = \mu_0(1 + 0.001i)$. Calculate the fractional power loss over a distance of one wavelength. (15%)

5. Assume that $\mathbf{J}(t) = \frac{1}{2}\,\mathrm{Re}[\mathbf{J}_0\exp(-i\omega t)]$ and $\mathrm{E}(t) = \mathrm{Re}[\mathrm{E}_0\exp(-i\omega t)]$, where $\mathbf{J}_0$ and $\mathbf{E}_0$ are complex constants and ω is a real constant. Show that the time-averaged value of $\mathbf{J}(t)\cdot\mathbf{E}(t)$ is given by $\mathrm{Re}[\mathbf{J}_0\cdot\mathbf{E}_0^*]$. (15%)

6. Verify Poynting's theorem for the case of a long, straight conducting wire of radius a and conductivity σ, which carries a direct current I. [The resistance per unit length of the wire is $R = 1/(\sigma\pi a^2)$] (20%)