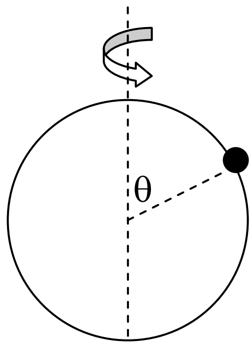
Classical Mechanics (2013)

1. There is a car moving with constant speed v on a horizontal circular orbit of radius R. A simple pendulum is suspended under the roof of the car. The length of the pendulum is L. Calculate the frequency of the pendulum.

(20 points)

2. A particle of mass m is constrained to move on a vertical hoop (circle) of radius R which is rotating around its vertical axis with a constant angular velocity ω. (a) Write down the Lagrangian including the gravitation field. (b) Write down the canonical momenta and Hamiltonian. (c) What physical quantities are conserved?



(30 points)

3. For a Hamiltonian system with coordinates q_i, momenta p_i and angular momentum **L**,
(a) show that the Poisson bracket $[q_i, p_j] = \delta_{ij}$, $[q_i, L_j] = \varepsilon_{ijk}q_k$ and $[p_i, L_j] = \varepsilon_{ijk}p_k$.
(b) A scalar F is a function of q_i and p_i: $F = F(q^2, p^2, q\cdot p)$. Show that $[F, L_j] = 0$.

(20 points)

4 The Lagrangian of a particle of mass m and charge e moving in magnetic field is $L = mv^2/2 + e\vec{A}\cdot\vec{v}/c$. (a) Derive the canonical momenta and Hamiltonian. (b) If the magnetic field $\vec{B}$ is uniform and along $\hat{z}$-direction, and $\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$, show that the Hamiltonian can be written as $H = \frac{p^2}{2m} - \omega_L l_z + \frac{m\omega_L{}^2}{2}r^2$ where $\omega_L = \frac{eB}{2mc}$ is the Larmor frequency. (c) Start from the Lagrangian of a simple harmonic oscillator in a inertia frame, then transform it to a rotating coordinate system, write down the Hamiltonian in the rotating coordinate system. Show that it has the same form as that in part (b). Note that the rotating coordinates (x',y') have the relations $x = x'\cos\omega t - y'\sin\omega t$ and $y = x'\sin\omega t + y'\cos\omega t$ where ω is the angular frequency of the rotating coordinates.

(30 points)