Quantum Mechanics II – Ph. D. Qualify Exam. 2013

(In spherical coordinates, $\nabla^2 = (1/r^2)\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + (1/r^2)[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + (\frac{1}{\sin\theta})^2\partial^2/\partial\square^2])$

1. Using the WKB method, derive the quantization conditions for the energy levels of a particle of mass m moving in (a) a linear potential $V(x) = \beta|x|$, $\beta > 0$, (b) the oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$.(20 pts)

2. (a) The Hamiltonian of a system of 3 **nonidentical** spin ½ particles is given by $H = -\varepsilon_0(\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3)/\hbar^2$ (ε_0 is a constant). Find the system's energy levels and their degeneracies. (b) Find the Clebsch-Gorden coefficients associated with the addition of two angular momenta $j_1 = 1$ and $j_2 = 1$. (20 pts)

3. (a) Use the variational method to estimate the ground state energy of a particle of mass m in a potential $V(x) = kx^4$, $k > 0$. (b) Calculate the energy shift in the ground state and in the degenerate 1st excited state of a 2-dimensional harmonic oscillator $H = (P_x^2 + P_y^2)/2m + \frac{1}{2}m\omega^2(x^2 + y^2)$ due to the perturbation $V = 2\lambda xy$. (20 pts)

4. (a) Please calculate the differential cross section $d\sigma/d\Omega$ and total cross section σ for the *Yukawa potential* $V(r) = g\exp(-\mu_0 r)/r$. (g and μ_0 are constants) (b) Please calculate the phase shift δ_l for a hard sphere, represented by $V(r) = \infty$, $r < r_0$ and $V(r) = 0$, $r > r_0$. (20 pts)

5. (a) For a localized electron, one can treat the spin as the only degree of freedom that the electron possesses. The Hamiltonian of such electron is $H = -\mathbf{M}\cdot\mathbf{B} = \frac{eg\hbar}{4mc}\,\boldsymbol{\sigma}\cdot\mathbf{B}$, where $\mathbf{B} = (0, 0, B)$. Suppose at t = 0 the spin is an eigenstate of S_x with eigenvalue $+\frac{1}{2}\hbar$, what are $\langle S_x \rangle$ and $\langle S_y \rangle$ at a later time t? ($\mathbf{S} = \frac{1}{2}\hbar\boldsymbol{\sigma}$, $\boldsymbol{\sigma}$ are the Pauli matrices) (b) Please write down the Dirac equation explicitly for a free particle with mass m and describe briefly the derivation processes. (20 pts)