Statistical Physics

(1.) Consider the three-dimensional black-body radiation of a photon gas. Show that the Helmholtz free energy $F = 8\pi V \frac{(kT)^4}{(hc)^3} \int_0^\infty x^2 \ln[1 - \exp(-x)]dx$. (20%)

(2.) Consider a free Fermi gas in two dimensions, confined to a square area $A = L^2$. Find the Fermi energy and the density of states. Explain how the chemical potential should behave as a function of temperature. (20%)

(3.) Find out the entropy of a diatomic oxygen gas near room temperature with N, V, m, and ϵ denoting the molecular number, the size of the container, the molecular mass, and the energy scale for rotational excitation, respectively. (20%)

(4.) For the paramagnetic material, the z component of a particle's magnetic moment is $\mu_z = -j\delta, (-j+1)\delta, \ldots, (j-1)\delta, j\delta$. In the presence of a magnetic field B pointing in the z direction, find out the total magnetization of a system of N such particles at temperature T. (20%)

(5.) Consider a classical monatomic ideal gas at a height z above sea level, so each molecule has potential energy mgz in addition to its kinetic energy. Find out the chemical potential of the system with N, V, m, and T denoting the molecular number, the size of the container, the molecular mass, and the temperature, respectively. (20%)