Classical Mechanics

1. A spherical pendulum is a mass particle attached to a massless rigid rod. The other end of the rod is fixed. Hence, the particle can move on a spherical surface. Write down the Lagrangian and Lagrange equations. (25%)

2. A particle with charge Q and mass M is moving on a plane. There is uniform magnetic field B perpendicular to the plane. (a) Write down the Lagrangian and Hamiltonian. (b) What are your constants of motion? (c) Solve the equations of motion. (25%)

3. $[A, B]$ is the Poisson bracket. If $\varphi(\vec{r}, \vec{p})$ is an isotropic function then $[\varphi, L_z]=0$ where L_z is the z-component of the angular momentum. (25%)

Hint: $\varphi(\vec{r}, \vec{p})$ can only be a function of r^2, p^2, and $\vec{r}\bullet\vec{p}$.

4. Consider an infinite chain of particles of mass m. Two adjacent particles are connected by a uniform and massless spring of length a and spring constant k. Assume there is no gravitation and the particles can only move along the chain. (a) Write down the Lagrangian and equations of motion. (b) In the continuum limit a ,m, $1/k \rightarrow 0$ but m/a and ka remain finite, write down the Lagrangian and equations of motion. (c) Find the solutions of the equations of motion in the continuum limit. (25%)