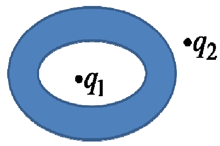
Classical Electrodynamics (I) PhD Qualifying Exam (6 problems)

Note: 1. This is a closed-book exam.
2. Terms and notations follow Jackson.

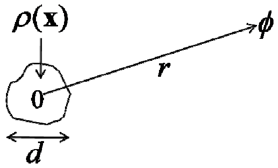
1. Consider a hollow, isolated conductor of arbitrary shape in free space. There is no net charge on the conductor. A static point charge q_1 is in the hollow region of the conductor. A static point charge q_2 is outside the conductor (see figure).



(a) Explain qualitatively how q_2 experiences a force due to the presence of q_1. (6%)
(b) Explain qualitatively how q_2 experiences an additional force due to q_2 itself. (6%)
(c) Explain qualitatively why q_1 experiences no force due to the presence of q_2. (6%)
(d) q_2 experiences a force due to the presence of q_1, but q_1 experiences no force due to the presence of q_2. Is Newton's third law (action-reaction law) violated? If your answer is yes, no explanation is needed. If your answer is no, explain qualitatively how the action-reaction law works here. (6%)

2. A straight wire of infinite length and radius a carries a cuurent I, which is uniformly distributed over its cross section. Find the field $\mathbf{B}$ both inside and outside the wire. (10%)

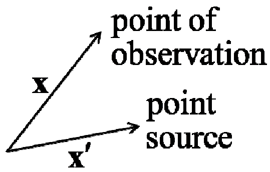
3. A static charge distribution $\rho(\mathbf{x})$ of dimension d (see figure) can be represented by multipole moments (monopole, dipole, quadruploe, etc.).



(a) Write down the expression for the dipole moment $\mathbf{p}$ with respect to the origin ($\mathbf{x} = 0$). (5%)
(b) At a distance r from the origin, how does the electric potential ϕ scale with r if $r \gg d$? (5%)

4. If a closed loop is formed entirely of a wire of infinite conductivity, then the electric field in the wire must be zero, or $\oint \mathbf{E} \cdot d\ell = 0$ around the loop. Now, a time-varying magnetic field is applied through the loop. Will $\oint \mathbf{E} \cdot d\ell$ remain 0? If your answer is no, no explanation is needed. If your answer is yes, explain how it can remain 0. (10%)

5. A singal emitted by a point source at position $\mathbf{x}'$ and time t' reaches a stationary observation point at position $\mathbf{x}$ and time t (see figure). Write the (retarded time) t' in terms of t, $\mathbf{x}$, $\mathbf{x}'$, and the speed of light c for the following two cases:



(a) The source is stationary at time t'. (8%)
(b) The source is moving toward the observation point at the speed of $c/2$ at time t'. (8%)

6. (a) Write down the four macroscopic Maxwell equations. (10%)
(b) Consider a plane electromagnetic wave of frequency ω and propagation constant $\mathbf{k}$ in an infinite and uniform medium of conductivity σ, electric permittivity ε, and magnetic permeability μ. Derive the dispersion relation (i.e. the relation between ω and k). (10%)
[vector formula: $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$]
(c) Assume σ, ε,.and μ are all real numbers. From the result in (b), derive the expression for the skin depth (δ) under the "good conductor" condition (i.e. the displacement current is negligible as compared with the conduction current). (10%)