1. A 1-dimensional simple harmonic oscillator (with angular frequency ω) is initially (at $t = 0$) in a state with wave function

$$\psi(x, 0) = N \sum_{n=0}^{\infty} c^n \psi_n$$

where ψ_n are the harmonic oscillator energy eigenfunctions and c is a complex parameter.

   (a) Evaluate the normalization constant N and calculate the probability of finding the system again in the initial state at a later time $t > 0$. **(8 pts)**

   (b) Compute the expectation value of energy $\langle H \rangle$. **(7 pts)**

2. The sodium nucleus has spin 3/2 and a magnetic moment $\boldsymbol{\mu}_{Na} = (g_{Na}e/2m_p)\mathbf{S}$, where the gyromagnetic ratio is $g_{Na} = 1.48$. The sodium nucleus is placed in a constant magnetic field in the z-direction $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$.

   (a) Calculate the first-order energy shifts if a perturbative magnetic field $\mathbf{B}' = B_1\hat{\mathbf{z}}$ is applied to the system. **(10 pts)**

   (b) If the perturbative magnetic field $\mathbf{B}' = B_2\hat{\mathbf{x}}$, calculate the second-order energy shifts. **(15 pts)**

3. A particle on a sphere is in the state $\psi(\theta, \phi) = \sqrt{\frac{15}{16\pi}}\ \sin 2\theta \cos\phi$.

   What are the probabilities of energy (H) and angular momentum ($\mathbf{L}^2$ and L_z) measurements? **(20 pts)**

4. (a) In scattering problem ($rV(r) \rightarrow 0$ as $r \rightarrow \infty$), we write $\Psi_{\mathbf{k}} = e^{ikz} + \Psi_{sc}(r, \theta, \phi)$, please show that $\Psi_{sc} \rightarrow \frac{e^{ikr}}{r} f(\theta, \phi)$ as $r \rightarrow \infty$ and $d\sigma/d\Omega = |f(\theta, \phi)|^2$. **(10 pts)**

   (b) From the transition rate, one obtain $d\sigma/d\Omega = (2\pi)^4 m^2 \hbar^2 |\langle \mathbf{p}_f | V(r') | \mathbf{p}_i \rangle|^2$, Assuming elastic scattering, please calculate the differential cross section $d\sigma/d\Omega$ and total cross section σ for the *Yukawa potential* $V(r) = g \exp(-\mu_0 r)/r$. ($g$ and μ_0 are constants). **(15 pts)**

5. Please calculate the phase shift δ_l for a hard sphere, represented by $V(r) = \infty$, $r < r_0$ and $V(r) = 0$, $r > r_0$. **(15 pts)**

First few spherical harmonics:

$$Y_0^0 = \sqrt{\frac{1}{4\pi}},$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} cos\theta \ , \ Y_1^{\pm1} = \mp\sqrt{\frac{3}{8\pi}} sin\theta e^{\pm i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1), Y_2^{\pm1} = \mp\sqrt{\frac{15}{8\pi}} sin\theta\ cos\theta e^{\pm i\phi}$$

$$Y_2^{\pm2} = \sqrt{\frac{15}{32}} \sin^2\theta e^{\pm 2i\phi}$$