

## Qualifying Exam of Galactic Astrophysics (2022)

1. (25%) As for the spiral galaxy,
  - (a) Please derive the Tully-Fisher relation. (15%)
  - (b) Use the mass-to-light ratio  $M/L$  to convert the mass to a luminosity and then find a relation between luminosity, circular velocity, central surface brightness, and  $M/L$  by assuming  $r = r_R$ . This is a generalized Tully–Fisher relation. (10%)
2. (25%) Please give a simple and clear description on Jean’s equation of a collision-less system.
3. (30 %) The Jeans instability can be analyzed exactly in rotating systems, without invoking the Jeans swindle. Consider a homogeneous, self-gravitating, barotropic fluid of density  $\rho_0$ , contained in an infinite cylinder of radius  $R_0$  whose symmetry axis is the  $z$  axis. The cylinder walls and the fluid rotate at angular speed  $\Omega = \Omega \hat{e}_z$ .
  - (a) Show that the gravitational field inside the cylinder is (10%)
$$-\nabla\Phi_0 = -2\pi G\rho_0 (x\hat{e}_x + y\hat{e}_y).$$
  - (b) Using Euler’s equation in a rotating frame, find the condition on  $\Omega$  so that the fluid is in equilibrium. (10%)
  - (c) Please describe the physical explanation following the thought of Toomre (1964). (10%)
4. (20%) Explain why the first star in the universe is likely to be quite massive.