Qualifying Exam Solid State Physics 2024

1. **Crystal symmetry (35pts)**

Someone tells you he found a crystal with the following lattice vectors:

$$a_1 = \left(\frac{a}{2}\right)\hat{x} + \left(\frac{a}{2}\right)\hat{y}, \quad a_2 = a\hat{y}, \qquad a_3 = \left(\frac{a}{\sqrt{2}}\right)\hat{z}$$

, where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of a Cartesian coordinate system.

1. Identify the Bravais lattice (drawing a picture may help)
2. Calculate the volume of the primitive cell.
3. Calculate the reciprocal lattice
4. Draw the 1st Brillouin zone

2. **Atomic Packing (25pts)**

Consider the hcp structure that is produced by stacking layers of equal-size spheres.

1. What is the ration between unit cell height and width?
2. What is the atomic packing factor?
3. If every second layer of spheres was half of the other layer's radius, what would be the packing factor?

3. **Crystals (15pts)**

LiCl molecules have a atomic separation $a = 5.14$Å .
1. What is the binding energy of a molecule after it crystallizes into the Sodium chloride crystal structure whose Madelung constant is $\alpha \approx 1.7476$?

4. **Electrons in solids (15pts)**

It should not be surprising by now to know that in some cases, a band may be anisotropic, i.e., as one goes from k = 0 in different directions in the 1st BZ, the curvature can be different. Consider a two-dimensional anisotropic band of the form

$$E(k) = \hbar^2\left[\frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2}\right]$$

1. What is the density of states when $m_1 = 2 \, m_2$

5. **Phonons (10pts)**

The phonon dispersion relation of a harmonic chain has a maximum possible frequency ω_{max}.
1. Find an equation that describes the propagation of a wave whose frequency is higher than ω_{max}.
2. What does your result mean?

Useful constants: $\varepsilon_0 = 8.85 \times 10^{-12} F/m$, 1Å $= 10^{-10}m$, $e = 1.602 \times 10^{-19}C$, $1eV = 1.602 \times 10^{-19}J$