Qualifying Exam - Introduction to Solid State Physics (2023)

This exam is closed-book. Please make sure that you put your name on all of your answer sheets and try to answer each question including their sub-questions.

Useful constants: $\varepsilon_0 = 8.85 \times 10^{-12} F/m$, $1\text{Å} = 10^{-10} m, e = 1.602 \times 10^{-19} C, 1eV = 1.602 \times 10^{-19} J, m_e = 9.1 \times 10^{-31} kg, \hbar = \frac{h}{2\pi} = 1 \times 10^{-34} Js$

Crystal structure (30%),

1. Graphene is a two-dimensional lattice which is built by a continuous arrangement of regular hexagons of carbon atoms. The side length of a hexagon is 0.14 nm.
    a. How many atoms are there in a primitive unit cell?
    b. Write down the basis and conventional lattice vectors.
    c. Sketch the reciprocal lattice and indicate the first Brillouin zone?
2. Show that the maximum proportion of the available volume which can be filled by hard spheres arranged on a simple cubic lattice is 0.52
3. Explain how the Madelung constant can be derived for a 1D ionic chain.

Diffraction (25%)

4. When electrons, X-rays, neutrons, or helium atoms are diffracted by a crystal, the same diffraction peaks are observed in each case. However, the intensities of the peaks depend on the type of beam that is diffracted. Why do all of these beams produce the same diffraction peaks and why are the intensities different for the different beams?
5. The diffraction condition can be expressed as Δk = G. Explain what a reciprocal lattice vector G is. Derive Bragg's law of diffraction from the diffraction condition.

Phonons (20%)

6. The speed of sound in a certain linear monatomic chain is 1.08×10^4 m/s. If the mass of each atom is $6.1 \times 10^{-26} kg$ and the atomic separation at equilibrium is 0.485 nm, find
    a. the effective spring constant
    b. the maximum normal mode angular frequency.

Electrons (25%)

7. Calculate the density of states for a one-dimensional chain of atoms with length L assuming two different dispersion relations
    a. E~k
    b. $E \sim k^2$
8. An n-type semiconductor is called “degenerate” if the Fermi energy is inside the conduction band. Considering the effective mass of ZnO $m_e^* = 0.27 m_e$ is ZnO with an electron concentration of $n = 10^{20} cm^{-3}$degenerate?