Classical Electrodynamics I

* *All quantities are expressed in terms of SI units.*

* *Vectors are represented with right-pointing arrow notation above their names, as in $\vec{v}$.*

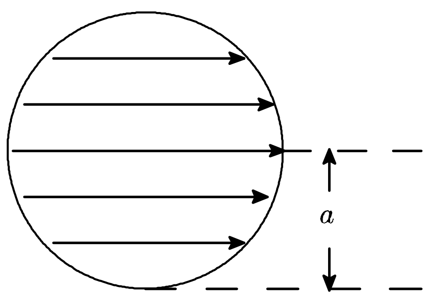
1. [20%] By altering the dielectric properties,

$$\epsilon(\vec{x}) \to \epsilon(\vec{x}) + \delta\epsilon(\vec{x})$$

The electrostatic energy of a linear medium is changed. Prove that the energy change δW_Q for fixed sources ($\delta\rho = 0$) is the negative of the energy change δW_V held at fixed potentials ($\delta\phi = 0$).

$$\delta W_Q = -\delta W_V$$

2. For a sphere of radius a, with a uniform permanent magnetization $\vec{M}$, embedded in a non-permeable medium,



(a) [15%] show that the magnetic fields inside the sphere due to the magnetization $\vec{M}$ are

$$\vec{H}_{in} = -\frac{1}{3}\vec{M}, \quad \vec{B}_{in} = \frac{2\mu_0}{3}\vec{M}$$

(b) [10%] If the magnetized sphere is in a uniform external magnetic field $\vec{B}_0 = \mu_0\vec{H}_0$ throughout all space, show that

$$\vec{B}_{in} + 2\mu_0\vec{H}_{in} = 3\vec{B}_0$$

3. [20%] Imagine a system of N distinct current-carrying circuits, the ith one with current I_i in otherwise empty space. For $i \neq j$, define the mutual inductance M_{ij} by

$$M_{ij} = \frac{1}{I_j}F_{ij},$$

where F_{ij} is the magnetic flux from circuit j linked within circuit i. Prove that M_{ij} are symmetric in i and j.

$$M_{ij} = M_{ji}$$

4. .

(a) [5%] What are the four macroscopic Maxwell equations (The equations that relate fields $\vec{E}$, $\vec{B}$, $\vec{D}$, $\vec{H}$ and sources ρ_f, $\vec{J}_f$)?

(b) [5%] Denote the microscopic electric and magnetic fields by $\vec{e}$ and $\vec{b}$. Let η and $\vec{j}$ be the microscopic charge and current densities. What are the four microscopic Maxwell equations (The equations that relate fields $\vec{e}$, $\vec{b}$ and sources η, $\vec{j}$)?

(c) [15%] By definition, $\vec{E} = \langle\vec{e}\rangle$ and $\vec{B} = \left\langle\vec{b}\right\rangle$: the macroscopic $\vec{E}$ and $\vec{B}$ are the average of $\vec{e}$ and $\vec{b}$, respectively. The average of a function $F(\vec{x}, t)$ with respect to a test function $f(\vec{x})$ is defined as

$$\langle F(\vec{x}, t)\rangle = \int f(\vec{x}')\, F(\vec{x} - \vec{x}', t)\, d^3x'$$

where $f(\vec{x})$ is real, nonzero in some neighborhood of $\vec{x} = 0$, and normalized to unity over all space. Is the macroscopic (free) charge density ρ_f equal to the average of the microscopic charge density η? Why?

5. [10%] A magnetostatic field is due entirely to a localized distribution of permanent magnetization. Show that

$$\int \vec{B} \cdot \vec{H} d^3x = 0$$

provided that the integral is taken over all space.