2014 Ph. D. Qualifying Exam — Quantum Mechanics II

1. {25%} The operator $H = \dfrac{[\vec{\sigma} \cdot (\vec{p} - \frac{e}{c}\vec{A})]^2}{2m}$ can be shown to be the sum of two operators, $\dfrac{(\vec{p} - \frac{e}{c}\vec{A})^2}{2m}$ and X, *i.e.*,

$$H = \frac{\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2}{2m} + X,$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, $\vec{p}$ is the momentum operator, and $\vec{A}$ is the vector potential. What is X?

2. {25%} A free particle of mass m, traveling with momentum p parallel to the z-axis, scatters off the potential

$$V(\vec{r}) = V_0 \left[\delta(\vec{r} - a\hat{z}) - \delta(\vec{r} + a\hat{z})\right].$$

Calculate the differential cross section, $\dfrac{d\sigma}{d\Omega}$, in the Born approximation.

3. {25%} Consider a one-dimensional harmonic oscillator governed by the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2.$$

If the oscillator is in the ground state for $t < 0$. For $t \geq 0$ it is subjected to a time-dependent but spatially uniform force in the x-direction,

$$F(t) = F_0 e^{-t/\tau}.$$

Obtain the probability of finding the oscillator in its first excited state for $t > 0$ by using time-dependent perturbation theory to first order. You may use

$$\langle n'|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\,\delta_{n',n+1} + \sqrt{n}\,\delta_{n',n-1}\right).$$

4. {25%} Please explain *two* of the following three terms:

   (a) Lamb shift

   (b) Aharonov-Bohn effect

   (c) Hyperfine splitting in Hydrogen atom