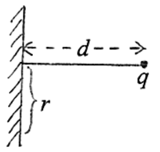
Classical Electrodynamics (I) PhD Qualifying Exam (5 problems)

Note: 1. This is a closed-book exam.
2. Terms and notations follow Jackson.

[1] A point charge q is situated a distance d from an infinite, flat, and grounded conducting surface (see figure below). Find the charge density σ on the conducting surface as a function of charge q, and distances d and r, where d and $r$are indicated in the figure. (10%)



[2] If a closed loop is formed entirely of a wire of infinite conductivity, then the electric field in the wire must be zero, or $\oint \mathbf{E}\cdot d\ell = 0$ around the loop. Now, a time-varying magnetic field is applied through the loop. Will $\oint \mathbf{E}\cdot d\ell$ remain 0? If your answer is no, no explanation is needed. If your answer is yes, explain how it can remain 0. (10%)

[3] Two identical, perfectly conducting loops are far apart and share the same axis. Each has self-inductance L and a current I flowing in the same direction, so the energy in each loop is $\frac{1}{2}LI^2$. They are then brought together and superposed.

(a) What is the final current in each loop? (10%)

[Hint: the magnetic flux through a perfectly conducting loop is unchanged.]

(b) What is work done in bringing the two loops together? Is the work done on the loops or by the loops? (10%)

[4] (a) Write down the 4 Maxwell equations in free space in the presence of $\mathbf{J}$ and ρ. (5%),

(b) Derive the wave equation for $\mathbf{B}(\mathbf{x}, t)$ in the absence of $\mathbf{J}$ and ρ. (5%)

(c) From the wave equation in (b), derive the relation between the frequency ω and the propagation constant k for a plane electromagnetic wave. (5%)

[vector formula: $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla \bullet \mathbf{A}) - \nabla^2\mathbf{A}$]

[5]Answer “yes” or “no” to the following statements (no explanation is required). (45%, 3 points for each correct answer; –1 point for each wrong answer; 0 point for each unanswered question. If total points <0, it will be regarded as 0.)

(1) In electrostatics, the potential ϕ is continuous and electric field $\mathbf{E}$ is discontinuous across a single layer of surface charge distribution.

(2) In electrostatics, ϕ is discontinuous and $\mathbf{E}$ is continuous across a dipole layer.

(3) The static electric field $\mathbf{E}$ can be calculated by using $\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$ alone.

(4) The integral form of Gauss’s law for $\mathbf{E}$ is *mathematically* equivalent to the differential form of Gauss’s law for $\mathbf{E}$ whether $\mathbf{E}$ is static or time-dependent.

(5) If $\oint_S \mathbf{A}(\mathbf{x})\cdot\mathbf{n}da = 0$ for any *closed* surface S (da is a differential area of the surface and $\mathbf{n}$ is its outward normal), it implies $\mathbf{A}(\mathbf{x}) = 0$ everywhere.

(6) The electric field E on the surface of a conductor with static surface charge density σ is $E=\sigma/\varepsilon_0$ even if the surface is curved or σ is non-uniform.

(7) $\varepsilon_0 E^2(\mathbf{x})$ and $q\delta(\mathbf{x}-\mathbf{x}_0)\Phi(\mathbf{x})$ have the same dimension

(8) Electric polarization **P** and electric dipole moment **p** do not have the same dimension.

(9) The polarization charge density ($\rho_{pol} = -\nabla\cdot\mathbf{P}$) is only a convenient definition. It does not represent real charges.

(10) The reason a 2.45 GHz microwave oven can heat food is because water molecules in food have a resonant frequency at approximately 2.45 GHz.

(11) The dipole moment **p** of a distribution of electrical charges is always independent of its point of reference.

(12) The magnetic field energy density, $w_b = \mathbf{H}\cdot\mathbf{B}$, is derived for a *linear* medium, i.e. $\mathbf{B} = \mu\mathbf{H}$ with μ independent of **B**.

(13) The magnetization current density ($\mathbf{J}_M = \nabla\times\mathbf{M}$) is only a convenient definition. It does not represent a real current.

(14) A static magnetic field can penetrate through a good conductor with $\mu = \mu_0$ as if there were no conductor present.

(15) Newton's law states that the total mechanical momentum of an isolated system is conserved. This is also true in electrodynamics.