Quantum Mechanics II -Qualify Exam, 2015

Problem 1. (20 points)

a) Express x, y and z in terms of spherical harmonic functions $Y_l^m(\theta, \phi)$ and $r = \sqrt{x^2 + y^2 + z^2}$. ($Y_0^0 = (1/4\pi)^{1/2}$, $Y_1^0 = (3/4\pi)^{1/2} \cos\theta$, $Y^{\pm}1_1 = \mp(3/8\pi)^{1/2} \sin\theta e^{\pm i\phi}$).

b) Is the wave function function $\psi = (a^5/6\pi)^{1/2}(x + y + 2z)e^{-ar}$ an eigen-function of $\vec{L}^2$? What are the probabilities of finding $l = 1$, and $m = \pm 1, 0$ for this wave function?

Problem 2. (30 points)

The spin states: $|s = 1, m = -1\rangle$ and $|s = 2, m = -1\rangle$ composed of spin-3/2 and spin-1/2 states are linear combinations of $|s_1 = 3/2, m_1 = -3/2; s_2 = 1/2, m_2 = 1/2\rangle$ and $|s_1 = 3/2, m_1 = -1/2; s_2 = 1/2, m_2 = -1/2\rangle$, that is

$|1, -1\rangle = \cos\theta |3/2, -3/2; 1/2, 1/2\rangle - \sin\theta |3/2, -1/2; 1/2, -1/2\rangle$,

$|2, -1\rangle = \sin\theta |3/2, -3/2; 1/2, 1/2\rangle + \cos\theta |3/2, -1/2; 1/2, -1/2\rangle$.

a) Determine the values for $\cos\theta$ and $\sin\theta$.

b) Express $|3/2, -3/2; 1/2, 1/2\rangle$ and $|3/2, -1/2; 1/2, -1/2\rangle$ as functions of $|1, -1\rangle$ and $2, -1\rangle$.

c) A system of two particles with spins $s_1 = 3/2$ and $s_2 = 1/2$ is described by the Hamiltonian $H = \alpha \vec{S}_1 \cdot \vec{S}_2 + \beta \vec{S}_1^2 + \gamma \vec{S}_2^2$, where α, β and γ are constants. Show that the $|2, -1>$ and $|1, -1>$ above are eigen-functions of H. Find the eigenvalues.

d) If the system is initially (at t=0) in the state: $|3/2, -1/2; 1/2. - 1/2\rangle$, find the state of the system at times $t > 0$. What is the probability of finding the system in the state $|3/2, -3/2; 1/2, 1/2\rangle$.

Problem 3. (25 points)

a) Write down the time-dependent first order correction due to a perturbation $H^1(t)$ for the probability amplitude d_f of the f-th unperturbed state when apply the perturbation at t =0 on a n-th unperturbed state.

b) Using results from a) to calculated the first order correction for non-zero d_f to the unperturbed wave function $|n^0\rangle$ of a Harmonic oscillator due to a time-dependent and x-dependent perturbation $H^1(t) = -cx(e^{-t/\tau_1} - e^{-t/\tau_2})$, where c is a constant.

c) For $n^0 = 2$, find the non-zero transition amplitudes to other states at time t at infinity.

Problem 4. (25 points)

Two identical spint-1 particles can form total spin-1 and spin-2 states with the spatial wave functions, $\psi(\vec{r}_1, \vec{r}_2)_1$ for spin-1, and $\psi(\vec{r}_1, \vec{r}_2)_2$ for spin-2, respectively.

a) Determine the sign changes for the total spin-1 and spin-2 spatial wave functions after exchangeing the two particle positions.

b) Assuming that the scattering amplitude is $f(\theta)$, find the values for $f(\pi/2)$ for the state with the total spin equal to spin-1. Assuming the interaction potential between the two identical particles in total spin-1 wave is $V(r) = (g/r)e^{-\mu r}$, and $f(\theta)$ is of the form $(-m/2\pi\hbar^2) \int (e^{-i\vec{q}\cdot\vec{r}} V(r) d^3r + CC)$ (CC is determined by the requirement that the amplitude is anti-symmetric when exchange the two identical particle positions), find $f(\theta)$ and obtain $d\sigma/d\Omega$.