Classical Electrodynamics (I) PhD Qualifying Exam (5 problems)

Note: 1. This is a closed-book exam. Things such as lecture notes, dictionary, calculator, and cell phone are NOT allowed.
2. No one can sit side by side with you.
3. Terms and notations follow Jackson.
4. If you use a unit system different from SI, state what it is

1. Consider an infinitesimally-thin layer in free space with static surface charge density σ as shown in the right figure.
   (a) Calculate the difference between the electric fields (**E**) on the two sides of the layer. (10%)
   Hint: Consider separately $\mathbf{E}_\perp$ and $\mathbf{E}_{||}$ (i.e. the components of **E** perpendicular and parallel to the layer).
   (b) Is the electrostatic potential ϕ continuous across the layer? Give your reason. (6%).

2. A cylindrical metal wire of infinite length and constant radius a carries a DC cuurent I. The axis of the wire coincides with the z-axis of a cylindrical coordinate system (r, θ, z) and the current flows in the z-direction.
   (a) Assume that the current is uniformly distributed over the cross-section of the wire. Calculate the vector magnetic force per unit volume on the free (or conduction) electrons of the wire in terms of I, a, and the radial position r ($\leq$ a). (8%)
   (b) At the steady state, the magnetic force on the free electrons at any r is balanced by an electric force. Explain qualitatively how the electric field is formed. (6%)
   (c) If I is sufficiently large, the wire will shrink and collapse despite the force balance on the free electrons. Explain qualitatively why this can happen. (6%)

3. Work needs to be done to move a permanet magnet at a constant velocity in the neighborhood of a conductor. Explain this effect in the context of the electromagnetic theory. (16%)

4. The fields of a plane wave in a dielectric medium have the spatial and time dependence of $e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}$. Consider the following two plane waves. Plane Wave 1 : $\mathbf{k} = (k_r + ik_i)\mathbf{e}_x$ and Plane Wave 2 : $\mathbf{k} = k_x\mathbf{e}_x + ik_z\mathbf{e}_z$, where ω, k_x, k_z, k_r, and k_i are positive real numbers, $\mathbf{e}_x$ and $\mathbf{e}_z$ are real unit vectors, and $i = \sqrt{-1}$.
   (a) What's the direction of propagation of each wave? (6%)
   (b) What's the main difference between these two waves? (6%)
   (c) Why are both waves called plane waves? (8%)

5. Consider Plane Wave 1 in Problem 4 again [i.e. $e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}$ with $\mathbf{k} = (k_r + ik_i)\mathbf{e}_x$]. Assume the dielectric medium is characterized by permeability μ and permittivity ε, where μ is real and positive, while ε is in general a complex number and is frequency dependent.
   (a) What is the physical origin for the imaginary part of ε? (6%)
   (b) What is the physical origin for the frequency dependence of ε? (6%)
   (c) Let $k = k_r + ik_i$. Write down the relation between ω and k (no derivation is required). (6%)
   (d) Let $\varepsilon = \varepsilon' + i\varepsilon''$ with $\varepsilon' > 0$, $\varepsilon'' > 0$, and $\varepsilon' \gg \varepsilon''$. Express the approximate phase velocity of the wave in terms of μ and ε'. Express the approximate field attenuation constant of the wave in terms of μ, ε', ε'', and ω. (10%)
   Hint: Apply binominal expansion to $\sqrt{\varepsilon}$ up to order $\varepsilon''/\varepsilon'$ to separate the real and imaginary parts of $\sqrt{\varepsilon}$.