

Classical Electrodynamics (I) PhD Qualifying Exam (5 problems)

Note: 1. This is a closed-book exam. Things such as lecture notes, dictionary, calculator, and cell phone are NOT allowed.

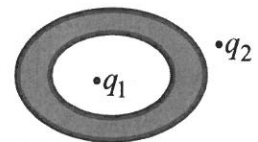
2. No one can sit side by side with you.

3. Terms and notations follow Jackson.

4. If you use a unit system different from SI, state what it is

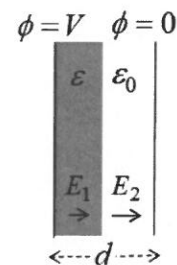
1. Consider a hollow, isolated conductor of arbitrary shape in free space.

There is no net charge on the conductor. A static point charge q_1 is in the hollow region of the conductor. A static point charge q_2 is outside the conductor (see figure).



- Explain qualitatively how q_2 experiences a force due to the presence of q_1 .
- Explain qualitatively how q_2 experiences an additional force due to q_2 itself.
- Explain qualitatively why q_1 experiences no force due to the presence of q_2 .
- q_2 experiences a force due to the presence of q_1 , but q_1 experiences no force due to the presence of q_2 . Is Newton's third law (action-reaction law) violated? If your answer is yes, no explanation is needed. If your answer is no, explain qualitatively how the action-reaction law works here.

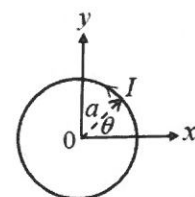
2. A capacitor is formed of two large parallel plate separated by a small distance d . Half of the space between the plates is filled with a dielectric of uniform permittivity ϵ ; the other half is free space (see figure). The capacitor is charged to a voltage V . Assume the electric field E_1 in the dielectric and E_2 in free space are both uniform and perpendicular to the plates.



- Write down the boundary condition between the dielectric and free space.
- Calculate E_1 and E_2 .

3. The magnetic dipole moment (\mathbf{m}) of a distributed current source [$\mathbf{J}(\mathbf{x})$] is defined by $\mathbf{m} = \frac{1}{2} \int \mathbf{x} \times \mathbf{J}(\mathbf{x}) d^3x$. A thin circular wire of radius a on the x - y plane carries a current I (see figure). [Note: $\mathbf{J}(\mathbf{x})$ is in A/m^2 and I is in A].

Do the following problems in cylindrical coordinates (r, θ, z) with the z -axis being the axis of the current loop and pointing out of the paper.



- Express $\mathbf{J}(\mathbf{x})$ in terms of I (hint: use delta functions).
- Write the position vector \mathbf{x} in cylindrical coordinates.
- Use $\mathbf{m} = \frac{1}{2} \int \mathbf{x} \times \mathbf{J}(\mathbf{x}) d^3x$ to calculate \mathbf{m} of the current loop.

- Write down the four microscopic Maxwell equations in differential form.
 - Show that conservation of charge is implicit in this set of equations.
 - Show that the electric field \mathbf{E} and magnetic induction \mathbf{B} can be represented by a scalar potential ϕ and a vector potential \mathbf{A} .
5. A plane electromagnetic wave with instantaneous fields \mathbf{E}_i and \mathbf{B}_i is incident normally from the free space onto a stationary and perfectly-conducting plane. The reflected wave has instantaneous fields \mathbf{E}_r and \mathbf{B}_r .
- What is the relation between \mathbf{E}_r and \mathbf{E}_i on the conductor? Give your reason.
 - What is the relation between \mathbf{B}_r and \mathbf{B}_i on the conductor? Give your reason.