

Qualifying Exam - Introduction to Solid State Physics (2021)

This exam is closed-book. Please make sure to answer each question and make sure to add all steps necessary.

Useful constants: $\epsilon = 8.85 \times \frac{10^{-12} \text{F}}{\text{m}}$, $1\text{\AA} = 10^{-10} \text{m}$, $e = 1.602 \times 10^{-19} \text{C}$, $1\text{eV} = 1.602 \times 10^{-19} \text{J}$, $m_e = 9.1 \times 10^{-31} \text{kg}$

Crystal structure

The hexagonally closed packing (hcp) structure represents one of the most dense ways to pack a monoatomic crystal.

- Can you imagine another crystal packing structure with an equally high density? (5%)
- How many percent of the hcp crystal volume remain unfilled? (10%)
- What is the ratio of hcp lattice constants in plane (a) and out-of-plane (c)? (5%)
- In graphite the ratio of c/a is approximately 2.4, what does that tell you about the difference in mechanical properties in-plane and out-of-plane? (5%)

Diffraction

Diffraction originates from the interference of waves that are reflected from a crystal.

- Explain what is meant by “Lattice Constant” for a cubic crystal structure. (5%)
- Explain why X-ray diffraction may be observed in first order from the (110) planes of a crystal with a body-centered cubic lattice, but not from the (110) planes of a crystal with a face-centered cubic lattice. (10%)
- A wave of wavelength $\lambda = 0.084 \text{nm}$ diffracts from a NaCl crystal and the lowest observed angle of diffraction is at 8.35° . What is the lattice spacing of this FCC crystal? (5%)
- What is the highest order of diffraction that can be observed in the experiment described in (c)? (5%)

Phonons

Imagine a crystal being subjected to sound pressure on one side.

- Plot the vibration amplitude for atoms at different position in the crystal (use the harmonic oscillator simplification) (5%)
- What is the highest frequency of sound that can be transmitted through a monoatomic crystal with a given spring constant and atomic mass? (5%)
- How would the vibration amplitude at different positions in the crystal change if the frequency of sound is higher than the maximum frequency, calculated in b)? (You can sketch the distribution, calculate it, or describe it in words). (10%)
- Describe the microscopic process of thermal expansion and explain why it is called an “anharmonic” process. (5%)

Electrons

- Imagine that the concentration of electrons within a metal was somehow doubled. What would be the change of the average electron energy? (10%)
- Explain the origin of the forbidden energy gap in insulators. (5%)
- An insulating material absorbs all wavelengths shorter than λ_0 . What is the width of the forbidden energy gap for this material? (5%)
- An electron near the top of the valence band in a semiconductor has energy $E = -10^{-37}|k|^2$ where E is in Joules and k is in m^{-1} . An electron is removed from a state $k = 2 \times 10^8 \text{m}^{-1} \hat{x}$, Calculate the effective mass and velocity of the resulting hole. (5%)