1. Consider a particle with spin quantum number s = 1. Ignore all spatial degrees of freedom and assume the particle is subjected to an external magnetic field $\vec{\mathbf{B}} = B\hat{x}$. The Hamiltonian operator of the system is $H = g\vec{\mathbf{B}}\cdot\vec{\mathbf{S}}$.
   (a) Obtain explicitly the spin matrices in the basis of the $\vec{\mathbf{S}}^2$, $\mathbf{S}_z$ eigenstates, $|s, m_s\rangle$. (8 pts)
   (b) If the particle is initially (at t = 0) in the state $|1, 1\rangle$, find the evolved state of the particle at times $t > 0$. (9 pts)
   (c) What is the probability of finding the particle in the state $|1, -1\rangle$? (8pts)
2. Please calculate the differential cross section $d\sigma/d\Omega$ and total cross section σ for
   (a) the *Yukawa potential* $V(r) = g \exp(-\mu_0 r)/r$. ($g$ and μ_0 are constants) (10 pts)
   (b) the Gaussian potential $V(r) = V_0 \exp(-r^2/r_0^2)$. (15 pts)
3. Consider the hydrogen atom, and assume that the proton, instead of being a point-source of the Coulomb field, is a uniformly charged sphere of radius $R << a_0$.
   (a) What is the resulting electrostatic potential $V_R(r)$? (5 pts)

   The difference $\Delta V(\mathrm{r}) = V_R(r) - (-\frac{e^2}{r})$ will be proportional to the assumed extension R of the radius.
   (b) Considering $\Delta V(r)$ as a perturbation, calculate the energy shift ΔE for n = 1, l = 0 state to first order. (8 pts)
   (c) Do the same for 2s and 2p states. (12 pts)
   (Note: $\Psi_{100}(r) = (\pi a_0^3)^{-1/2} e^{-r/a_0}$; $\psi_{200}(r) = (32\pi a_0^3)^{-1/2}(2 - r/a_0)e^{-r/2a_0}$
   $\psi_{210}(r) = (32\pi a_0^3)^{-1/2}(r/a_0)e^{-r/2a_0}\cos\theta$

   $$I_n(\lambda) = \int_0^\lambda dx\, x^n e^{-x} = n!\,(1 - e^{-\lambda}\sum_{\nu=0}^{n}\frac{\lambda^\nu}{\nu!})$$)
4. A particle of mass m in a potential $V(x) = \frac{1}{2}m\omega^2x^2$. If the relativistic expression of the kinetic energy is used, what is the shift in the ground state energy? (11 pts)
   ($\boldsymbol{a} = \sqrt{\frac{mw}{2\hbar}}\boldsymbol{X} + i\sqrt{\frac{1}{2m\ \hbar}}\boldsymbol{P}$; $\boldsymbol{a}^+ = \sqrt{\frac{mw}{2\hbar}}\boldsymbol{X} - i\sqrt{\frac{1}{2mw\hbar}}\boldsymbol{P}$)
5. A particle of mass m is initially in the ground state (E_1) of an infinite square well of width L. Starting at time t = 0, the system is subjected to the perturbation $H'(t) = V_0 x e^{-\alpha t^2}$, where V_0 and α are constants.
   (a) Find the probability that the energy is measured to be E_2 in the limit $t \to \infty$. (7 pts)
   (b) Find the probability that the energy is measured to be E_3 in the limit $t \to \infty$. (7 pts)