Qualifying exam: classical electrodynamics

Note: (1) This is a closed-book exam. Notes, dictionary, calculator, and cell phone are NOT allowed.
(2) No one can sit side-by-side with you.
(3) Terms and notations follow the textbook of J. D. Jackson, if not mentioned additionally.
(4) If answers have units different from the SI, please describe it explicitly.

1. Electrostatics

(a) One electric charge $+3q$ is located on the z-axis at $z = -2d$, with $d > 0$. Write down the electrostatic potential Φ and the electric field $\boldsymbol{E}$ in all space. (10%)

(b) In the half space of $z \leq 0$ there is an ideal conductor. Right above its surface at $z = 0$ there is the electric field $\boldsymbol{E}(x,y) = E_z(x,y)\boldsymbol{e_z}$ with $\boldsymbol{e_z}$ as the unit vector along the z axis. $E_z(x,y)$ is a function of the x and y coordinates on the surface. Derive the surface charge density $\sigma(x,y)$ on the surface. (10%)

(c) An ideally conducting and grounded spherically shell with an inner diameter of r and an outer diameter of R is centered at the origin of the coordinates. Within its hollow interior space there is a charge $-q$ at its center. Write down the electrostatic potential Φ and electric field $\boldsymbol{E}$ outside the shell. (10%)

2. Electric dipole and dielectrics

(a) Following **1(a)**, there is another electric charge $-3q$ located on the z-axis at $z = 2d$. Write down first the exact electrostatic potential Φ in all space. To derive the potential far away from these charges Φ_{dipole}, expand Φ as a power series of d and derive the potential in first-order of d while keeping qd as a finite quantity. Write down the final answer of Φ_{dipole} as well as the associated electric field $\boldsymbol{E}_{\text{dipole}}$. (15%)

(b) Following **1(b)**, the conductor is now replaced by an isolated, linear dielectric with an electrical susceptibility of ε. Write down the surface charge density $\sigma(x,y)$ on its surface induced by E_z. (5%)

3. Magnetostatics

(a) There is a z-independent magnetic field $\boldsymbol{H}(x,y) = H_0[-y/(x^2+y^2)]\boldsymbol{e_x}+H_0[x/(x^2+y^2)]\boldsymbol{e_y}$ in the three-dimensional space. Here $\boldsymbol{e_x}$ and as $\boldsymbol{e_y}$ are the unit vectors along the x- and y-axis, and H_0 is a constant. Derive the current density $\boldsymbol{J}(x,y)$ at any position except that on the z-axis. (5%)

(b) Calculate the magnitude and the direction of the total current in **3(a)** along the z-axis. (10%)

(c) A magnetic material with a linear magnetic permeability μ is located at $x = y = a$ in the magnetic field $\boldsymbol{H}$ in **3(a)**. Derive the magnetization $\boldsymbol{M}$ induced by $\boldsymbol{H}$. (5%)

4. Maxwell equations

(a) Write down the Maxwell equations with the electric field $\boldsymbol{E}$ and the magnetic flux density $\boldsymbol{B}$ with charge and current density ρ and $\boldsymbol{J}$ in vacuum in differential form. (12%)

(b) By using two of the equations in **4(a)** derive the continuity equation. (6%)

(c) From **4(a)**, use the divergence theorem to derive two of the Maxwell equations in integral form. (6%)

(d) Following **4(c)**, derive the other two Maxwell equations in integral form via the Stokes' theorem with the curl of $\boldsymbol{E}$ and $\boldsymbol{B}$. (6%)