Full Frontal Physics: Naked Black Hole Firewalls

陳丕燊教授跨國團隊《赤裸的黑洞火牆》發表於

Physical Review Letters 期刊

這聽起來可能像是一本很糟的言情小說，但物理學家卻用「赤裸」（naked）、「無戲劇性」（no-drama）這一類的形容詞，來企圖描述宇宙間最炙手可熱的話題，那就是人類認知的宇宙中最奇怪的物體—黑洞。

Artist's concept illustrating a supermassive black hole with millions to billions times the mass of our sun (Image: NASA/JPL-Caltech)

過去四年裡，物理學家在研究黑洞物理的數學結構時，提出了一個怪異的想法：黑洞其實應該有一道「火牆」（firewall），任何東西如果企圖「翻牆」進入黑洞，就會被它徹底摧毀。然而，一篇發表在著名的《物理論壇通訊》(Physical Review Letters)的新文章《赤裸的黑洞火牆》（Naked Black Hold Firewalls），對這個觀點提出了質疑。該論文的共同作者，國立台灣大學物理系陳丕燊教授解釋，這篇文章的目的，是想用一個普遍接受的量子微擾概念，去攻擊那些導致「火牆悖論」背後的黑洞物理基本信條。

陳丕燊教授的共同作者包括北歐理論物理研究中心（Nordita）研究員王元君（Yen Chin Ong）博士（他於 2114 年獲得台大天文物理博士）、加拿大阿爾伯塔大學（University of Alberta）佩舉（Don Page）教授、

京都大學佐佐木節（Misao Sasaki）教授，及台大梁次震宇宙學中心研究員廉東翰（Dong-han Yeom)博士。

The five authors of the paper with another colleague during the discussion at the Yukawa Institute for Theoretical Physics: (L to R) Dong-han Yeom, Yen Chin Ong, Pisin Chen, Don Page, Yasusada Nambu, and Misao Sasaki.

圖片：五個作者由左至右為：廉東翰、王元君、陳丕燊、佩舉（Don Page）、Yasusada Nambu（非作者）、以及佐佐木節（Misao Sasaki）。 照片攝於由陳丕燊召開並主持之京都大學 YITP「黑洞信息遺失悖論」研討會。

人類對於黑洞的概念來自愛因斯坦的相對論：一個使時空扭曲的巨大物體，當它裡面有了足夠多的物質，就會使這個區域的時空變得非常陡峭，連光都無法逃離。既然連光都無法逃離，這些物體就是我們所稱的黑洞。它們也是人類可理解的最大宇宙垃圾處理場。如果一個不幸的時空旅行者進入了黑洞的「事件視界」（event horizon），根據這個模型，他就會被完全摧毀在黑洞裡。

先不說這是明顯的簡化，一直以來黑洞總是迷人卻難以描述。1970 年代霍京（Stephen Hawking）發現黑洞並非全黑，實際上某些粒子可以透過量子纏結而產生，從黑洞中逃離，這理論就是著名的霍京輻射。自此以後，黑洞物理的領域就成了各種奇異現象的泉源，需要量子理論與相對論

相結合的數學才能完整解釋。在黑洞物理過去紛擾的 40 年中，有一個突出而未決的問題—「黑洞資訊遺失悖論」，這也是霍京首先提出的。這個問題持續阻礙物理學者直接以量子理論的數學切入解釋。佩舉解釋：「一開始，多數致力於愛因斯坦重力理論的學者認為霍京原本的建議是對的。就是說，資訊在黑洞形成和蒸發時就已經遺失了。」佩舉是第一個在 1990 年代提出重要論文反對霍京建議的人。他表示：「現在，包括霍金本人的多數重力物理學家都相信資訊並沒有遺失。然而，黑洞資訊如何保存的這個部分，依然是個謎。」

在量子力學中，基於量子「確定性」和「可逆性」這兩個原則，信息在任何物理過程中，始終被保留。然而，由於物質帶著訊息落入黑洞，在穿越事件視界的過程中，物質中的信息顯然被消滅了一段時間，這個表面上的矛盾一直困擾的物理學家。這個悖論本身的出現就是導因於霍輻射。這表明物質可以從黑洞輻射出，但是分析顯示，在輻射的初期並沒有大量攜帶出原先掉進黑洞的信息。在 2012 年，一群物理學家研究這個悖論時發現，導致這一悖論的三個基本假設不可能都是一致的。也就是說，「統一性」與「局部量子場論」這兩個基本原則與「無戲劇性」的假設相互矛盾。這後者假設是根據愛因思坦的廣義相對論。它意味著，當一個物體通過事件視界應該沒有任何異狀發生。為了解決這個矛盾，他們提出了他們認為最保守的解決方案。那就是，在黑洞的表面上確實有「火牆」的存在，它會焚化任何一個落進黑洞的物體。這個主張是令人相當吃驚的，因為一個足夠大的黑洞,例如馬中佩教授最近發現的超大黑洞,它的曲率幾乎是微乎其微，所以廣義相對論應該可以完全適用而不需要加入量子場論。因此我們期待任何物質在穿越視界時應該順利通關而不應該被強制焚化。共同作者國立台灣大學陳丕燊教授說：「所謂的火牆，指的是在我們所認知的黑洞的表面，有一個高能量密度區域，它會破壞任何墜入的東西。」

陳教授於2015年輪休，赴京都大學「湯川理論物理研究所」(Yukawa Institute for Theoretical Physics)擔任訪問學者時，召開了一個國際小型研討會(Molecular-Type Workshop)，邀請幾位國際黑洞物理專家研究「黑洞信息遺失悖論」。這篇《赤裸的黑洞火牆》論文就是這個研討會的結晶。作者們論證，基於量子力學，霍京輻射時必然不可免的會衍生隨機的量子微擾。而黑洞本身對於這些量子微擾的反作用，必將使火牆脫離黑洞的事件視界而裸露在外，能夠被遙遠的觀察者看到。

共同作者京都大學佐佐木節教授說：「如果火牆的確存在，它不僅能將一個下落的物體摧毀，甚至可從外面看見物體被焚毀。」這篇論文強調，像這樣存在於事件視界之外的「赤裸的火牆（naked firewall）」的概念是有問題的。如果火牆確實存在，作者們質疑它不會輕易地被黑洞局限於一個區域內，而是赤裸在外。它的破壞力可能超越事件視界，遷移至可以被觀察到的空間。這使得火牆的概念相對大膽，並非倡議火牆的那幾位物理學家所說的那麼保守。《赤裸的黑洞火牆》作者們建議，物理學界須要再投入相當的努力去找尋「黑洞資訊遺失悖論」的最終答案。

此新聞原載於2016年4月21日「台大校訊」及網路焦點新聞：
http://www.ntu.edu.tw/spotlight/2016/794_20160422.html

其英文版原載於加拿大 University of Alberta 網站:
https://uofa.ualberta.ca/science/science-news/2016/april/full-frontal-physics

此篇論文內容詳載於下：

Naked Black Hole Firewalls

Pisin Chen,[1,2,*] Yen Chin Ong,[3,†] Don N. Page,[4,‡] Misao Sasaki,[5,§] and Dong-han Yeom[1,‖]
[1]*Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan*
[2]*Kavli Institute for Particle Astrophysics and Cosmology, SLAC National Accelerator Laboratory, Stanford University, California 94305, USA*
[3]*Nordita, KTH Royal Institute of Technology & Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden*
[4]*Theoretical Physics Institute, University of Alberta, Edmonton, Alberta T6G 2E1, Canada*
[5]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

(Received 25 November 2015; revised manuscript received 9 March 2016; published 22 April 2016)

In the firewall proposal, it is assumed that the firewall lies near the event horizon and should not be observable except by infalling observers, who are presumably terminated at the firewall. However, if the firewall is located near where the horizon would have been, based on the spacetime evolution up to that time, later quantum fluctuations of the Hawking emission rate can cause the "teleological" event horizon to have migrated to the inside of the firewall location, rendering the firewall naked. In principle, the firewall can be arbitrarily far outside the horizon. This casts doubt about the notion that firewalls are the "most conservative" solution to the information loss paradox.

DOI: 10.1103/PhysRevLett.116.161304

The black hole information loss paradox [1] is still unresolved almost 40 years after the issue was raised by Hawking. The debate was further heated by the firewall proposal raised by Almheiri, Marolf, Polchinski, and Sully (hereinafter, AMPS) [2]. See also AMPSS (short for AMPS, together with Stanford) for further arguments and clarifications [3]. Essentially, AMPS pointed out that local quantum field theory, unitarity, and no drama (the assumption that infalling observers should not experience anything unusual at the event horizon if the black hole is sufficiently large) cannot all be consistent with each other. Implicitly, it is also assumed that the Bekenstein-Hawking entropy corresponds to the statistical entropy of the black hole, which not everyone agrees—see Refs. [4,5] for recent reviews. Furthermore, it is assumed that there exists an observer who could collect all the Hawking radiation so as to attempt to violate the no-cloning theorem of quantum information by eventually falling into the black hole. AMPS argued that the "most conservative" resolution to this inherent inconsistency between the various assumptions (hereinafter, the AMPS paradox) is to give up no drama. Instead, an infalling observer would be terminated once he or she hits the so-called firewall. This seems rather surprising because the curvature is negligibly small at the event horizon of a sufficiently large black hole, and thus one would expect nothing special but low energy physics.

Essentially, the argument for a firewall is the following. Assuming unitarity, the information contained inside a black hole should eventually be recovered from the Hawking radiation. The information content is presumably encoded in the highly entangled Hawking radiation, and it is usually argued that the information should start to "leak out" after the black hole has lost approximately half of its initial Bekenstein-Hawking entropy, at the Page time [6–8]. A black hole that has passed its Page time is said to be "old," otherwise the black hole is considered "young." In other words, the late time radiation purifies the earlier radiation (which was emitted before the Page time and is—to a very good approximation—thermal). Thus, as the AMPS argument goes, the late time radiation is maximally entangled with the earlier radiation; and by the monogamy of quantum entanglement, the late time radiation cannot be maximally entangled with the interior of the black hole. This means that the field configuration across the event horizon is generically not continuous, which leads to a divergent local energy density. More explicitly, we recall that the quantum field Hamiltonian contains terms like $(\partial_r \varphi)^2$. The derivative is divergent at some $r = R$ if the field configuration is not continuous across R. This is the firewall. (See also Ref. [9], and p. 26 of Ref. [10].)

Usually it is thought that firewalls lie on the black hole event horizons. Of course in quantum mechanics there are no sharp boundaries, and the positions of event horizons should be uncertain, up to perhaps fluctuations of the order of the Planck length. That is to say, firewalls are presumably like stretched horizons [11], with the crucial difference that anything that hits a firewall gets incinerated instead of just passing right through, unharmed [3]. It is also possible that firewalls lie slightly *inside* the event horizons. In that case, a firewall would fall toward the (assumed spacelike) singularity (or whatever replaces the singularity in the quantum theory of gravity) faster than the black hole could shrink in size. However, supposedly a new firewall will be dynamically "replenished" on each fast-scrambling time scale [3]. (We shall restrict our attention to the asymptotically flat four-dimensional Schwarzschild black hole. The fast-scrambling time is of the order $M \log M$ [12], cf. the information retention time, which is of the order M^3.) By the nature of the event horizon, if the firewall lies either inside or exactly on the horizon, then it is completely invisible to the observers outside. For firewalls that are not too far outside the event horizons, it is still doubtful that they are perceptible to far-away observers, since it would seem that such firewalls are well hidden inside the Planckian region of the *local* thermal atmosphere. (The Hawking temperature is a quantity measured at infinity, but the local temperature near the horizon is enormously blueshifted to a trans-Planckian temperature, following the Tolman law. See, e.g., Ref. [13].)

Here we make the assumption that a firewall, if it exists, has a location determined by the past history of the Hawking evaporating black hole spacetime and is near where the event horizon would be if the evaporation rate were smooth, without quantum fluctuations. (If the firewall were far inside the event horizon, it would not resolve the paradox that it is proposed to resolve.) Then we show that quantum fluctuations of the evaporation rate in the future can migrate the event horizon to the inside of the firewall location, rendering it naked.

For simplicity, we shall approximate the metric near the horizon of an evaporating black hole by the Vaidya metric with a negative energy influx:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right)dv^2 + 2dvdr + r^2 d\Omega^2. \quad (1)$$

Here, $M(v)$ is the mass of the black hole, which is decreasing as a function of the advanced time v. For a smooth evaporation rate of a spherical black hole emitting mainly photons and gravitons, we shall take (in Planck units)

$$\dot{M} \equiv \frac{dM}{dv} = -\frac{\alpha}{M^2}, \quad (2)$$

where α is a constant that has been numerically evaluated [14–18] to be about 3.7474×10^{-5}.

The apparent horizon is located at $r_{\mathrm{ApH}} = 2M(v)$, whereas the event horizon is generated by radially outgoing null geodesics, which obey

$$\dot{r} \equiv \frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2M(v)}{r}\right), \quad (3)$$

and are on the boundary of such null geodesics reaching out to future null infinity, instead of falling in to the singularity that is believed to be inside the black hole. For a smooth evaporation rate given by Eq. (2), the event horizon is given by the solution to Eq. (3) such that it does not diverge exponentially far away from the apparent horizon in the future. If we define $u \equiv 1 - r/(2M)$ and $p \equiv -4\dot{M}$ and assume that $n \equiv -d \ln p / d \ln M$ is constant, then one can show that the event horizon is at

$$\begin{aligned} u = p &+ (n-2)p^2 + (n-1)(2n-5)p^3 \\ &+ (6n^3 - 28n^2 + 37n - 14)p^4 + O(p^5). \end{aligned} \quad (4)$$

For a smooth Hawking evaporation into massless particles with $p \equiv -4\dot{M} = 4\alpha/M^2$, so that $n = 2$, one finds that the event horizon is at

$$r_{\mathrm{EH}} = 2M[1 - 4\alpha/M^2 + O(\alpha^3/M^6)]. \quad (5)$$

For a general spherical metric, the covariant generalization of d/dv along an outward null direction toward the future is $d/dv \equiv N^\alpha \partial/\partial x^\alpha$, with outward null vector N^α normalized so that $\dot{r} \equiv dr/dv \equiv N^\alpha r_{,\alpha} = (1/2)\nabla r \cdot \nabla r = (1/2) - M/r$. Note that $dM/dv = -\alpha/M^2$ implies that $d^2(M^3)/dv^2 = 0$, but since $r_{\mathrm{EH}} \approx 2M$, we have $d^2(r_{\mathrm{EH}}^3)/dv^2 \approx 0$ as well. Let us therefore define an *adiabatic horizon* at r_{AdH} by the outer root of

$$\frac{d^2}{dv^2}(r_{\mathrm{AdH}}^3) \equiv N^\alpha \frac{\partial}{\partial x^\alpha}\left(N^\beta \frac{\partial}{\partial x^\beta} r_{\mathrm{AdH}}^3\right) = 0. \quad (6)$$

The location of the adiabatic horizon is very near where the event horizon would be if the future evolution of the latter followed the adiabatic mass evolution law of Eq. (2). One can show that r_{AdH} is equivalent to the location where the gradient vector of $(1/4)\nabla(r^2) \cdot \nabla(r^2) = r^2 \nabla r \cdot \nabla r \equiv r^2 - 2Mr$ (which, incidentally, defines M) is in the outward null direction, or $N^\alpha (r^2 \nabla r \cdot \nabla r)_{,\alpha} = 0$, which gives $\dot{M} = (1/2) - (3/2)(M/r) + (M/r)^2$ and

$$r_{\mathrm{AdH}} \equiv \frac{4M}{3 - \sqrt{1 + 16\dot{M}}}. \quad (7)$$

When Eq. (2) holds, this expression agrees with Eq. (5) to the order given.

We shall assume that the firewall, if it exists, is close to where the event horizon would be if the black hole evolved smoothly and adiabatically according to Eq. (2). However, the actual event horizon depends on the future evolution of the spacetime, and not just on that of its past. Therefore, quantum fluctuations in the future spacetime can lead the event horizon to deviate significantly from the unperturbed adiabatic horizon. If the mass loss rate exceeds the adiabatic formula, then the event horizon will be inside the adiabatic horizon. As a result, a firewall located at the adiabatic horizon would become naked, visible from future null infinity. See Fig. 1 for a diagrammatic explanation.

From Eq. (3), one can write the mass $M = M(v)$ in the Vaidya metric in terms of the event horizon radius $r = r(v) \equiv r_{\mathrm{EH}}(v)$ as

$$M = \frac{1}{2}r - r\dot{r}. \quad (8)$$

Let M_1, r_1 and M_2, r_2 be the unperturbed mass and radius of the black hole and their fluctuations, respectively, with total mass $M = M_1 + M_2$ and event horizon radius $r = r_1 + r_2$. (Note that we are comparing the true event horizon at $r = r_1 + r_2$ with where it would have been, at r_1, if there were no perturbation r_2, but this is not the same as

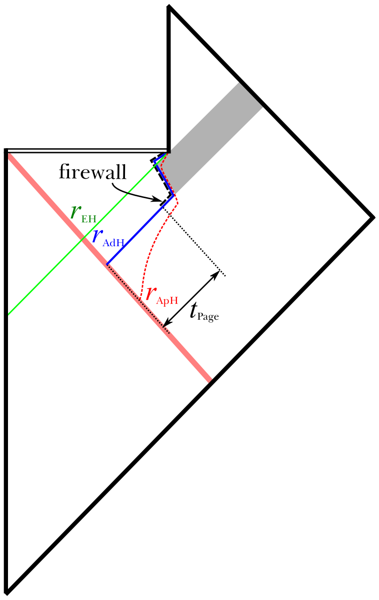

FIG. 1. A conceptual Penrose diagram illustrating the formation of a Schwarzschild black hole from a collapsing null shell, and its subsequent Hawking evaporation. Here, the event horizon ($r_{\rm EH}$) has been shifted inward some distance from the adiabatic horizon ($r_{\rm AdH}$) due to a quantum fluctuation. This renders the firewall (denoted by the dashed curve that appears after the Page time $t_{\rm Page}$) naked. The apparent horizon ($r_{\rm ApH}$) is also shown for comparison, but light rays can escape from inside it, since the black hole is shrinking.

what Eq. (7) would define as the "adiabatic horizon" when the perturbation is included. The adiabatic horizon would be very near the event horizon when the mass loss rate has a smooth form such as that given by Eq. (2), but for a significant perturbation $\dot{M}_2$ in the mass loss rate, the adiabatic horizon need not be near either the event horizon at r or the unperturbed horizon at r_1. We hence emphasize that the unperturbed horizon should not be confused with the adiabatic horizon once the mass perturbation M_2 becomes significant.) Now suppose that the unperturbed mass loss would give $M = M_1 = M_1(v) = (1/2)r_1 - r_1\dot{r}_1$, such that $\dot{M}_1 \approx -\alpha/M_1^2$, and that quantum fluctuations $M_2 = M_2(v)$ and $r_2 = r_2(v)$ are small compared with the total mass and the event horizon radius, respectively. Then,

$$\begin{aligned} M = M_1 + M_2 &= \frac{1}{2}r - r\dot{r} \\ &= \frac{1}{2}(r_1 + r_2) - (r_1 + r_2)(\dot{r}_1 + \dot{r}_2) \\ &\approx M_1 + \frac{1}{2}r_2 - r_1\dot{r}_2. \end{aligned} \tag{9}$$

For simplicity, we are making the highly idealized assumption that even with quantum fluctuations, the metric remains spherically symmetric and Vaidya near the event horizon, though this is not crucial for our argument.

Now for some particular advanced time $v = v_0$, let us ignore quantum fluctuations before this time, so that $M_2(v) = 0$ for $v < v_0$, and let us define the constant $M_0 = M(v_0) = M_1(v_0)$. To leading order in $M_0 \gg 1$ and $|v - v_0| \ll M_0^3$, the fractional decay of the black hole over the advanced time $v - v_0$ is small, and the negative of the coefficient of $\dot{r}_2$ in Eq. (9) may be written as $r_1 \approx 2M_1 \approx 2M_0$. Then, Eq. (9) gives $(1/2)r_2 - 2M_0\dot{r}_2 \approx M_2(v)$. The solution of this differential equation that is void of an exponentially growing departure of the event horizon $r(v) = r_1 + r_2$ from the unperturbed horizon $r_1(v)$ at late times is

$$r_2 \approx \exp\left(\frac{v - v_0}{4M_0}\right)\int_v^\infty dv' \frac{M_2(v')}{2M_0}\exp\left(\frac{v_0 - v'}{4M_0}\right). \tag{10}$$

Since the unperturbed evolution gives $\dot{M}_1 \approx -\alpha/M_0^2$ for $M_0 \gg 1$ and $|v - v_0| \ll M_0^3$, let us consider a quantum mass fluctuation that gives, with $\theta(v - v_0)$ the Heaviside step function,

$$\dot{M}_2 = -\theta(v - v_0)\frac{\alpha\beta}{M_0^2}\exp\left(-\frac{\gamma(v - v_0)}{4M_0}\right), \tag{11}$$

which has two new constant parameters, namely, β for how large the quantum fluctuation in the energy emission rate is relative to the unperturbed emission rate $-\alpha/M^2$ (with β assumed to be positive so that the quantum fluctuation increases the emission rate above the unperturbed value), and γ for how fast the quantum fluctuation in the energy emission rate decays over an advanced time of $4M_0$ (the inverse of the surface gravity κ of the black hole). Then with $M_2(v) = 0$ for $v < v_0$, one gets

$$M_2 \approx -\theta(v - v_0)\frac{4\alpha\beta}{\gamma M_0}\left[1 - \exp\left(-\frac{\gamma(v - v_0)}{4M_0}\right)\right]. \tag{12}$$

Plugging this back into Eq. (10) then gives

$$\begin{aligned} r_2 \approx &-\theta(v_0 - v)\frac{8\alpha\beta}{(1+\gamma)M_0}\exp\left(\frac{v - v_0}{4M_0}\right) \\ &-\theta(v - v_0)\frac{8\alpha\beta}{\gamma(1+\gamma)M_0}\left[1 + \gamma - \exp\left(-\frac{\gamma(v - v_0)}{4M_0}\right)\right]. \end{aligned} \tag{13}$$

This particular form of the emission rate fluctuation implies that the total mass fluctuation from the unperturbed evolution is $M_2(\infty) = -4\alpha\beta/(\gamma M_0)$. Then the radial fluctuation in the event horizon radius at the advanced time $v = v_0$, when $-r_2(v)$ has its maximum value, is

$$r_2(v_0) \approx \frac{2\gamma}{1+\gamma}M_2(\infty). \tag{14}$$

This means that if the quantum fluctuation in the energy emission rate is very short compared with $4M_0$ (decaying rapidly in comparison with the surface gravity of the black hole), so that $\gamma \gg 1$, then $r_2(v_0) \approx 2M_2(\infty)$, twice the total mass fluctuation. However, we shall just assume that γ is of the order of unity and hence get $r_2(v_0) \sim M_2(\infty)$ as an order-of-magnitude relation. Note that the reduction in the radius of the event horizon at $v = v_0$, where the fluctuation in the mass emission rate starts, occurs *before* there is any decrease in the mass below the unperturbed value $M_1(v)$, because the location of the "teleological" event horizon is defined by the future evolution of the spacetime.

Note that $r_1 \sim M_1 \sim M_0$, $r_2 \sim 1/M_0$, $\dot{r}_1 \sim 1/M_0$, and $\dot{r}_2 \sim 1/M_0^2$. This is consistent with the approximations made in Eq. (9) to drop the terms $r_2\dot{r}_2$ and $r_2\dot{r}_1$.

Therefore, if the putative firewall occurs at a location determined purely causally by the past behavior of the spacetime, and is sufficiently near where the event horizon would be under unperturbed adiabatic emission thereafter (say near the adiabatic horizon), then quantum fluctuations, at later advanced times that reduce the mass of the hole below that given by the unperturbed evolution, would move the actual event horizon inward (even before quantum fluctuations in the mass emission rate begin), so that the event horizon becomes inside the location of the putative firewall. That is, quantum fluctuations that increase the mass emission rate render such a firewall naked, visible to the external universe.

One possible objection to this conclusion is that for α, β, and γ all of the order of unity, the inward shift in the event horizon is by a change of radius, r_2, of the order of $1/M$, so that the proper distance from the putative firewall near $r = r_1$ to the event horizon at $r = r_1 + r_2$, in the frame of the timelike firewall surface outside the event horizon, is of the order of the Planck length. The proper acceleration of an observer that stays of the order of the Planck length outside the event horizon would be of the order of the Planck acceleration, giving an Unruh temperature of the order of the Planck temperature. One might object that quantum gravity effects at such extreme accelerations would make a naked firewall in practice indistinguishable from a firewall at or inside the event horizon.

However, for a black hole of huge initial entropy $S \gg 1$ that emits roughly S particles during its Hawking evaporation, there are a large number of roughly S approximately independent chances for the proper distance fluctuation of the event horizon inside the firewall to reach a large value, say $L \gg 1$, so that the probability at any one time needs only be $P(L) \sim 1/S$. For a large fluctuation L, the most probable way to produce it at $v = v_0$, when the Hawking temperature is $T_0 = 1/(8\pi M_0)$, is to have thereafter the radiation be locally thermal with a time-dependent temperature $T(v) = T_0(z+1)/[z+1-ze^{-(v-v_0)/(4M_0)}]$ with a constant parameter $z = [T(v_0) - T_0]/T_0 \gg 1$ chosen to give the desired $L = [8M_0(-r_2(v_0))]^{1/2}$. The probability of this fluctuation then works out to be $P(L) \sim \exp[-(\pi/2)L^2]$. Setting this to be $\sim 1/S$ then gives the most probable largest value of the fluctuation as $L \approx [(2/\pi)\ln S]^{1/2}$, which is arbitrarily large for arbitrarily large S. Therefore, arbitrarily large black holes can have the event horizon fluctuate an arbitrarily great distance inside a firewall whose location is determined causally. Hence, the firewall of an arbitrarily large black hole will with high probability become highly naked, observable without encountering quantum gravity effects (other than what quantum gravity effects are supposed to lead to the existence of the firewall itself).

Therefore, the firewall is *not* hidden in the region with Planckian local temperature; its presence would truly be at odds with expectations from general relativity and ordinary quantum field theory. More specifically, being in the exterior of the event horizon means that the firewall could potentially influence the exterior spacetime, so that even observers who do not fall into the black hole could have a fiery experience. In addition, the presence of a firewall well outside the event horizon could affect the spectrum of the Hawking radiation, which means that the presence of a firewall could be inferred even by asymptotic observers. Such a "naked firewall," i.e., a firewall far outside the event horizon, is therefore problematic, and giving up the no-drama assumption no longer seems like a palatable most conservative option.

A natural interpretation is that if there is a firewall, then it should affect not only the interior geometry of the black hole, but also the asymptotic future. The former would "only" violate general relativity for a free-falling observer, while the latter would violate the semiclassical quantum field theory for an asymptotic observer [19,20]. Thus, the firewall solution can hardly be considered as conservative.

We gratefully acknowledge the generous support and the stimulating environment provided by the Yukawa Institute for Theoretical Physics through its Molecule-type Workshop on "Black Hole Information Loss Paradox" (YITP-T-15-01), where the concept of this Letter was formulated. In addition, the work of P. C. is supported in part by Taiwan's National Center for Theoretical Sciences (NCTS) and Taiwan's Ministry of Science and Technology (MOST), of D. N. P. by the Natural Sciences and Engineering Council of Canada, of M. S. by MEXT KAKENHI Grant No. 15H05888, and of D. Y. by the Leung Center for Cosmology and Particle Astrophysics (LeCosPA) of National Taiwan University (103R4000). Y. C. O. thanks Nordita for various travel supports.

---

[*] pisinchen@phys.ntu.edu.tw
[†] yenchin.ong@nordita.org
[‡] profdonpage@gmail.com
[§] misao@yukawa.kyoto-u.ac.jp
[‖] innocent.yeom@gmail.com

[1] S. Hawking, Breakdown of predictability in gravitational collapse, Phys. Rev. D **14**, 2460 (1976).

[2] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, Black holes: Complementarity or firewalls?, J. High Energy Phys. 02 (2013) 062.

[3] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford, and J. Sully, An apologia for firewalls, J. High Energy Phys. 09 (2013) 018.

[4] P. Chen, Y. C. Ong, and D.-h. Yeom, Black hole remnants and the information loss paradox, Phys. Rep. **603**, 1 (2015).

[5] S. B. Giddings, Statistical physics of black holes as quantum-mechanical systems, Phys. Rev. D **88**, 104013 (2013).

[6] D. N. Page, Average Entropy of a Subsystem, Phys. Rev. Lett. **71**, 1291 (1993).

[7] D. N. Page, Information in Black Hole Radiation, Phys. Rev. Lett. **71**, 3743 (1993).

[8] D. N. Page, Time dependence of Hawking radiation entropy, J. Cosmol. Astropart. Phys. 09 (2013) 028.

[9] S. L. Braunstein, S. Pirandola, and K. Życzkowski, Better Late than Never: Information Retrieval from Black Holes, Phys. Rev. Lett. **110**, 101301 (2013).

[10] S. B. Giddings, "Quantum mechanics of black holes," in *Proceedings of Summer School in High-Energy Physics and Cosmology, Trieste, Italy, 1994*, edited by E. Gava, A. Masiero, K. S. Narain, S. Randjbar-Daemi, and Q. Shafi (World Scientific, River Edge, N.J., 1995), pp. 530–574; arXiv:hep-th/9412138.

[11] L. Susskind, L. Thorlacius, and J. Uglum, The stretched horizon and black hole complementarity, Phys. Rev. D **48**, 3743 (1993).

[12] P. Hayden and J. Preskill, Black holes as mirrors: Quantum information in random subsystems, J. High Energy Phys. 09 (2007) 120.

[13] R. M. Wald, Gravitation, thermodynamics, and quantum theory, Classical Quantum Gravity **16**, A177 (1999).

[14] D. N. Page, Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole, Phys. Rev. D **13**, 198 (1976).

[15] D. N. Page, Particle emission rates from a black hole. 2. Massless particles from a rotating hole, Phys. Rev. D **14**, 3260 (1976).

[16] D. N. Page, Particle emission rates from a black hole. 3. Charged leptons from a nonrotating hole, Phys. Rev. D **16**, 2402 (1977).

[17] D. N. Page, Comment on Entropy Evaporated by a Black Hole, Phys. Rev. Lett. **50**, 1013 (1983).

[18] D. N. Page, Hawking radiation and black hole thermodynamics, New J. Phys. **7**, 203 (2005).

[19] D.-i. Hwang, B.-H. Lee, and D.-h. Yeom, Is the firewall consistent?: Gedanken experiments on black hole complementarity and firewall proposal, J. Cosmol. Astropart. Phys. 01 (2013) 005.

[20] W. Kim, B.-H. Lee, and D.-h. Yeom, Black hole complementarity and firewall in two dimensions, J. High Energy Phys. 05 (2013) 060.