

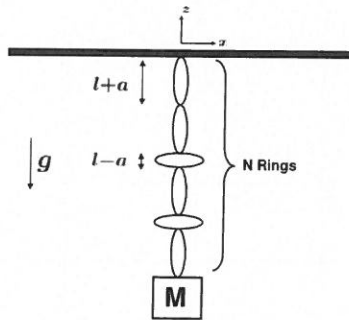
NTU-Physics Statistical Physics Qualifying Exam (2021)

Please note: 5 problems on 2 pages; Answers in both Chinese and English are OK.

1. (20%) A one dimensional chain is hung on a ceiling. One of its extremes is fixed, while the other holds a mass M . Gravity is acting along the negative z direction. The chain is formed by two kinds of distinguishable rings: they are ellipses with the major axis oriented vertically or horizontally. The major and minor axes have lengths $l+a$ and $l-a$ respectively. The number of rings is fixed to N and the chain is in thermal equilibrium at temperature T . Find the average energy U and the average length $\langle L \rangle$ of the chain. Finally, determine the linear response function ($F = Mg$)

$$\chi = \left(\frac{\partial \langle L \rangle}{\partial F} \right)_{T,N}$$

in the limit of high temperatures.



2. (20%) A three dimensional volume is separated in two parts by a rigid and impenetrable wall. The first part contains a Fermi gas composed of particles with spin $S = 1/2$, while the second one a Fermi gas of particles with spin $S = 3/2$. In both cases the single particle energy is $\epsilon = \frac{p^2}{2m}$, with p the absolute value of the momentum and m the associated mass. Determine the density ratio at the mechanical equilibrium in the limit of zero temperature.
3. (20%) Consider an ideal Bose gas with N particles confined to a two dimensional domain of area A at finite temperature T . Express the number of particles in the excited states, N_e , and the number of particles in the ground state, N_0 , in terms of $z = e^{\mu\beta}$, T , and A . Show that the system does not exhibit Bose-Einstein condensation unless $T \rightarrow 0$.
4. (20%) Given a two-level system in thermodynamic equilibrium with its environment at temperature T that is comparable with the energy gap Δ , calculate the entropy, average energy and the specific heat of the system. Explain how this system can have negative temperatures.

5. (20%) Figure below shows the low temperature specific heat measurements of two bulk materials. Explain what kind of information you can deduce from the data about these materials.

