量力(一)

1. (a) Consider a one-dimensional particle which is confined within the region $0 \le x \le L$ and whose wave function is $\psi(x, t) = \sin(\pi x/L)\exp(-i\omega t)$. Please find the potential $V(x)$. (10 pt) (b) The electron in the hydrogen is in a state described by $\frac{1}{4}[2\psi_{100}(\mathbf{r}) + 3\psi_{211}(\mathbf{r}) + \psi_{200}(\mathbf{r}) - \sqrt{2}\psi_{21\text{-}1}(\mathbf{r})]$. What are the expectation values of the **energy**, $\mathbf{L}^2$ and L_z? (15 pt.)
2. (a) Use the variational method to estimate the ground state energy of a particle of mass m in a potential $V(x) = kx^4$, $k > 0$. (10 pt) (b) Consider N (N >>> 1) non-interacting fermions inside an infinite potential well between $x = 0$ and $x = L$, what is the highest energy level that occupied by the fermions.(15 pt)
3. (a) A particle of mass m subjected to a potential $V(x) = -V_0\delta(x)$, where $V_0 > 0$. In the case of negative energies ($E < 0$), find the binding energy and the wave function. (15 pts) (b) Let $|n\rangle$ be an eigenstate of the 1-dim harmonic oscillator (1DHO) potential. Please calculate the expectation value of the operators $\boldsymbol{P}^2$ and $\boldsymbol{X}^3$ in the N-representation with the state $|n\rangle$ (i.e., $\langle n|\boldsymbol{P}^2|n\rangle$ and $\langle n|\boldsymbol{X}^3|n\rangle$). (10 pt)
4. (a) Consider a particle of mass m moving freely between $x = 0$ and $x = L$ inside an infinite square well potential. If the particle was in the ground state. Suddenly the right-hand side is moved to $x = 2L$. What is the probability that the particle will be in the ground state of the new potential? (10 pts) (b) There is a small potential $V(x) = \varepsilon\sin(\pi x/L)$ inside an infinite square potential $0 \le x \le L$. Please calculate the energy shifts for all excited states to first order in e due to $V(x)$. (15 pts)

The following information may (or may not) be useful.

$$\boldsymbol{a} = \sqrt{\frac{mw}{2h}}\boldsymbol{X} + i\sqrt{\frac{1}{2mwh}}\boldsymbol{P};\ \boldsymbol{a}^+ = \sqrt{\frac{mw}{2h}}\boldsymbol{X} - i\sqrt{\frac{1}{2mwh}}\boldsymbol{P})$$

$$H_n(x) = (-1)^n \exp(x^2)\, d^n \exp(-x^2)/dx^n$$

For hydrogen atom:

$$\psi_{100} = (\pi a_0^3)^{-1/2}\exp(-r/a_0)$$

$$\psi_{200} = (2a_0)^{-3/2}2(1 - r/2a_0)\exp(-r/2a_0)\sqrt{\frac{1}{4\pi}}$$

$$\psi_{21,1} = (2a_0)^{-3/2}\sqrt{\frac{1}{3}}\,(r/a_0)\exp(-r/2a_0)\,(-\sqrt{\frac{3}{8\pi}}e^{i\phi}\sin\theta)$$

$$\psi_{210} = (2a_0)^{-3/2}\sqrt{\frac{1}{3}}\,(r/a_0)\exp(-r/2a_0)\sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\psi_{21,-1} = (2a_0)^{-3/2}\sqrt{\frac{1}{3}}\,(r/a_0)\exp(-r/2a_0)\,(\sqrt{\frac{3}{8\pi}}e^{-i\phi}\sin\theta)$$