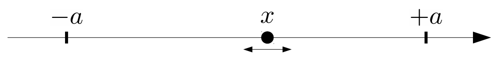
Qualification Exam

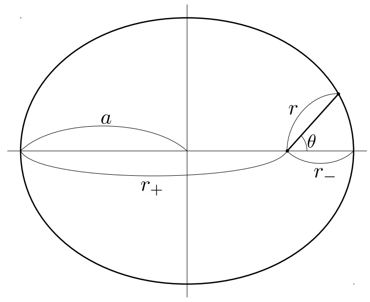
1. A point of mass m, charge e moves along the x-axis between two other points with the same charge e fixed at $x = \pm a$. The force between two charges separated at distance r obeys Coulomb's law $F = ke^2/r^2$.

(1) Write down the Lagrangian for the point.
(2) Find the frequency for the small oscillation of the point around its equilibrium position.
(25 points)

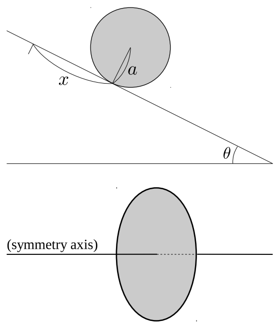


2. A point of mass m is orbiting around the source of attractive central force $F = k/r^2$.

(1) Write the Lagrangian for the motion in the orbital plane using polar coordinates r, θ.
(2) Express the energy E and angular momentum ℓ using r, θ and their time derivatives.
(3) Express $r_\pm$ (the maximum / minimum values of r) using k, m, E, ℓ.
(25 points)



3. Consider a disk of mass m, radius a and uniform mass density rolling down the slope of angle θ fixed in a vertical plane without slipping. Let g be the constant of gravitational acceleration.
(1) Find the moment of inertia of the disk about its symmetry axis.
(2) Use x for the position of the disk along the slope, and write down the Lagrangian. Then solve the equation of motion for x.
(25 points)



4. The polar coordinates r, θ are related to Cartesian coordinates x, y by

$$x = r\cos\theta,\ \ y = r\sin\theta.$$

Let us find the relation between two sets of canonical variables, (x, y, p_x, p_y) and $(r, \theta, p_r, p_\theta)$.

(1) Find the generating function $G(r, \theta, p_x, p_y)$ which satisfies

$$p_r dr + p_\theta d\theta = -x dp_x - y dp_y + dG\,.$$

(2) Express p_r, p_θ as functions of x, y, p_x and p_y.

(3) Show $[r, p_r] = 1$ using the Poisson brackets of (x, y, p_x, p_y).

(25 points)