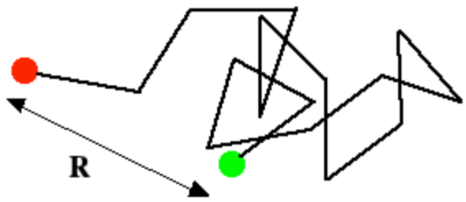
Qualifying Exam of Stellar Astrophysics 2020

1. Consider a path of random scattering $\vec{R} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \cdots \vec{r}_N$. N is the number of scattering. The figure is as follows.



Let $l_*{}^2 = Nl^2 = N\left(\frac{\langle R^2 \rangle}{N}\right), l_* = L$ and $\frac{L}{l} = \alpha L = \tau$. Please prove that $N \approx \tau^2$ when $\tau \gg 1$, that is, optically thick, $N \approx \tau$ when $\tau \ll 1$, that is, optically thin. Namely, $N \approx Max(\tau, \tau^2)$ or $N \approx \tau + \tau^2$. (20%)

2. Please find out the definition of *brightness temperature*, T_b, and under Rayleigh-Jeans limit (i.e. $h\nu \ll kT$), show that

$$T_b = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

(20%)

3. Please derive the Jeans condition for a gaseous, self-gravitating system with finite temperature by linearizing the equations of hydrodynamics. Meanwhile you should explain the criteria of the gravitational stability/instability. (20%)

P.S. The equations you may need to know in this problem:

The hydrodynamical equations of a self-gravitating system,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla \varphi - \frac{1}{\rho}\nabla P$$

$$\nabla^2 \varphi = 4\pi G \rho$$

$$c_s^2 = \frac{\partial P}{\partial \rho}$$

4. (a) Please explain the '*light house effect*' and the emission mechanism of a pulsar. (10%)

   (b) Please describe Faraday rotation and its application (10%)

5. Electron degeneracy pressure will halt the gravitational collapse of a star if its mass is below the *Chandrasekhar limit* (1.44 $M_{\odot}$). This is the pressure that prevents a *white dwarf* star from collapsing. Please use the dimension analysis to derive the equation of states under the cases of relativistic and non-relativistic. (Hint: $dE = E_k V d^3k/\pi^2$ in Fourier space) (20%)