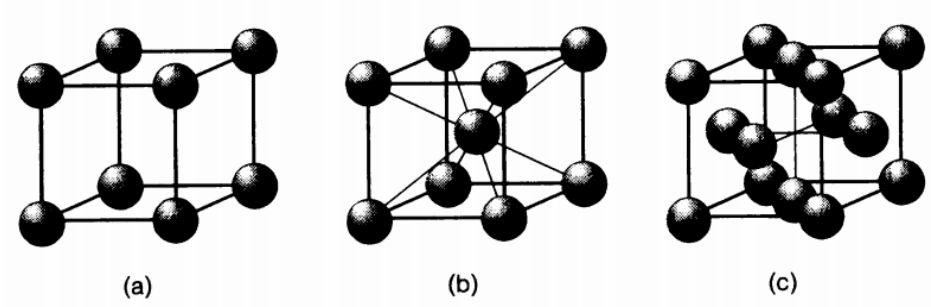
Qualifying Exam - Introduction to Solid State Physics (2019)

This exam is closed-book. Please make sure that you put your name on all of your answer sheets and try to answer each question including their sub-questions.

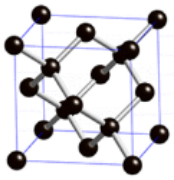
Useful constants: $\varepsilon = 8.85 \times 10^{-12} F/m$, $1\text{Å} = 10^{-10} m, e = 1.602 \times 10^{-19} C, 1eV = 1.602 \times 10^{-19} J, m_e = 9.1 \times 10^{-31} kg$

Crystal structure (25%)

1. Calculate the maximum portion of the volume that spherical atoms can fill (i.e. the filling factor) when arranged in the following three cubic lattices (a-c):



   a. Diamond exhibits the highest hardness of any material. Can this be explained by the packing density of its crystal structure (on the right)?



2. Estimate the energy that is gained when Li^+ and Cl^- are binding to form a LiCl molecule with separation a=5.14Å.
   a. Experiments find that the energy of LiCl in a crystal is approximately 75% higher than for an LiCl molecule, despite similar bond distances. Give an explanation why!

Diffraction (25%)

3. Diffraction originates from the interference of waves that are reflected from a crystal.
   a. Why do we only observe reflections with constructive interference (i.e. bright spots) and no dark spots?
   b. Consider diamond and zincblende, which crystallize in the same crystal structure but have different number of chemical species in their unit cell. Can you distinguish the materials by their diffraction spectra? If yes, how?
   c. Diffraction is an elastic process where the energy of the incident wave is transferred to the scattered wave. What would happen to the diffraction spectrum (peak position, number of peaks,…) if the energy in the scattering process was not conserved?
   d. List three possible modifications (i.e. change of band gap) to a crystal that you could detect as a change of their diffraction spectrum and describe the according change.

Phonons (25%)

4. A phonon arises from collective vibrations of neighboring atoms in a crystal.
    a. Plot the vibration amplitude for atoms in an ideal crystal (that exhibits harmonic oscillations between neighbors) that is subjected to sound pressure on one side.
    b. Under which amplitude distribution between atoms in the crystal will no sound be transmitted through the crystal?
    c. Can you use this mechanical phonon picture to explain why metals (one atom per unit cell) cannot transmit light?
    d. Why is a phonon called a quasi (i.e. almost)-particle while an electron is considered a particle? What are their similarities and differences?

Electrons (25%)

1. A "band-gap" is an energy region that separates valence band and conduction band in semiconductors and insulators.
    a. Describe briefly the origin of the band gap!
    b. What happens to an electron that is shot into such a material and exhibits an energy within the band-gap?
    c. How would the average energy of electrons change in a perfect crystal if the band gap was doubled?
2. The heat capacity of a material depends on its energy at a temperature and contains contributions from electrons and phonons.
    a. How would the average energy of electrons in a metal change when the temperature was increased a little bit starting from 0K? What if we started at room temperature?
    b. How will the ratio of phonon heat capacity to electron heat capacity change with temperature?