Introduction to Solid State Physics - NTU/Grad Inst Appl Phys – 博士資格考試 – 02/23-24/2013

[Total points 156]

1. [2+2+3+2+3+4+3 pts] A fundamental concept in the description of any crystalline solid is that of Bravais lattice, which specifies the periodic array in which the repeated units of the crystal are arranged.

   (a) What is a Bravais lattice? Define one.

   (b) How many distinctive two-dimensional lattice types, and what are they (and draw them)?

   (c) How many distinctive three-dimensional lattice types, and what are they (and draw them)?

   (d) What is a primitive cell?

   (e) Which one of the Bravais lattice is the primitive cell of bcc lattice? Calculate the angle and the lattice constant(s).

   (f) What are the volume (conventional cell), lattice points per conventional cell, volume of primitive cell, lattice points per unit cell, number of nearest neighbors, nearest-neighbor distance, and packing fraction of a face-centered cubic lattice?

   (g) Prove that in a cubic crystal a direction [*hkl*] is perpendicular to a plane (*hkl*) having the same indices. What is the distance between each successive plane?

2. [10 pts] What are Bragg and Von Laue formulations of x-ray diffraction by a crystal? Demonstrate the equivalence of the Bragg and Von Laue formulations.

3. [2+3+3+3+3 pts] (a) What is the structure factor in the x-ray diffraction from crystals? Calculate the structure factors for (b) simple cubic, (c) bcc, (d) fcc, and (e) diamond. Tabulate reflections which are possibly present in these lattices.

4. [5 pts] What are the principal types of crystalline binding? Give at least one example of material systems to each type.

5. [10 pts] Derive the van der Waals interaction either from consideration of two identical harmonic oscillators or considering electrostatics for the energy of interaction of two dipoles. Estimate the melting point of inert gas krypton with an atom to atom separation of 4 Å, using U (R) $\approx -10^{-58}$ erg-cm^6/R^6 and the Boltzmann constant k_B of 1.38 x 10^{-16} erg/K.

6. [4+7 pts] **Monatomic linear lattice.** Consider a longitudinal wave

$$u_s = u\,cos(\omega t - sKa)$$

   which propagates in a monatomic linear lattice of atoms mass M, spacing a, and nearest-neighbor interaction C.

   (a) Show that the total energy of the wave is

$$E = \frac{1}{2}M\sum_s\left(\frac{du_s}{dt}\right)^2 + \frac{1}{2}C\sum_s(u_s - u_{s+1})^2.$$

where s runs over all atoms.

(b) By substitution of u_s, in this expression, show that the time-average total energy per atom is

$$\frac{1}{4}M\omega^2u^2 + \frac{1}{2}C(1-cosKa)u^2 = \frac{1}{2}M\omega^2u^2$$

where in the last step we have used the dispersion relation below.

$$\omega^2 = (4C/M)sin^2\frac{1}{2}Ka \ , \ \omega = (4C/M)^{1/2}\left|sin\frac{1}{2}Ka\right|$$

7. [**3 pts**] What is the Debye model for (phonon) density of states? What is the Einstein model for (phonon) density of states?

8. [**2 pts**] On the specific heat of a classical crystal, what is the law of Dulong and Petit?

9. [**3+3+3+3 pts**] In the study of solids, particularly metals, there is a model called Drude Theory, which uses classical statistical mechanics. (a) Describe this theory as much as you can. (b) What are the major accomplishments and failures? (c) How did Sommerfeld modify the Drude Theory? (d) Again, what are the major accomplishments and failures of the Sommerfeld's modification?

10. [**2+10** pts] (a) What is Bloch's Theorem? (b) Prove it. (You may prove it from general quantum-mechanical consideration or other considerations.)

11. [**2+5+7 pts**] (a) What is the origin of the energy gap? (b) For a linear solid of lattice constant *a*, with a potential of U(x) = 2UcosGx, where G=2π/*a* is the reciprocal lattice, show that the magnitude of the energy gap at the Brillouin zone boundary is 2U (in the first order). (Extra points will be given if you also are able to derive the energy gap from the central equation: $(\lambda_k - \varepsilon)C(k) + \Sigma_G\ U_GC(k\text{-}G) = 0$, where $\lambda_k = h^2k^2/(2\pi)^2 2m$ and $H\psi(x) = (p^2/2m + U(x))\psi(x) = \varepsilon\psi(x)$. Assume that the Fourier component of the potential energy is small in comparison with the kinetic energy of a free electron at the zone boundary.)

12. [**3 pts**] What are "holes" in semiconductors? What are the (a) intrinsic and (b) extrinsic semiconductors?

13. [**3 pts**] Explain the physical limit of magnetic media for information storage.

14. [**10 pts**] For a group of *N* uniformly distributed, randomly oriented nano-magnets (each with magnetization *m*) in an external magnetic field *H*, assuming that the probability of direction of each nano-magnet is described by the Boltzmann factor $Exp(-U/k_BT)$, show that the total magnetization is described by

$M(H) = N\,m\,L(\alpha)$ $\quad \alpha = mH/k_BT$, where $L(\alpha) = coth\ \alpha - 1/\alpha$ is the Langevin function.

[Useful integration: $\int x^n e^{\alpha \cdot x} dx = \frac{x^n e^{\alpha \cdot x}}{\alpha} - \frac{n}{\alpha} \int x^{n-1} e^{\alpha \cdot x} dx$]

15. [**1 + 4 pts**] (a) Explain the acronyms of the following microscopy techniques: 1) STM; 2)AFM; 3)MFM. (b) To measure the effective tunneling barrier height, which of the following technique should be used; pick correct answers from each parenthesis, (I, V, Z) vs. (I, V, Z) spectroscopy, with Z feedback (ON, OFF), why?

16. [**3 pts**] What are the electrical and magnetic properties of a superconductor?

17. [**20 pts**] A multilayer made of N periods of D thick bilayers, which are consisted of material 1 with density r1 and thickness D1 and material 2 with density r2 and thickness D2, as illustrated by the schematic shown in Fig(a). Fig(b) illustrates the simulated x-ray reflectivity curve of a Mo(D1/ D = 1/3)/Si(D2/ D = 2/3) multilayer, in which every third reflectivity peak (marked by an arrow) is missing. Prove mathematically that every third reflectivity peak is missing.