Qualifying Exam - Introduction to Solid State Physics (2022)

This exam is closed-book. Please make sure that you put your name on all of your answer sheets and try to answer each question including their sub-questions.

Useful constants: $\varepsilon_0 = 8.85 \times 10^{-12} F/m$, $1Å = 10^{-10} m, e = 1.602 \times 10^{-19} C, 1eV = 1.602 \times 10^{-19} J, m_e = 9.1 \times 10^{-31} kg$

Crystal structure (20%)

1. Calcium Fluoride, CaF_2, has an fcc Bravais lattice and a basis with Ca at 000 and F's at fractional coordinates ¼ ¼ ¼ and ¾ ¾ ¾ of the conventional (cubic) unit cell. Sketch one conventional (cubic) unit cell of the structure. Sketch a primitive unit cell. The lattice constant is a= 5.451 Å. What is the distance from a Ca to an F atom in Angstroms?

Diffraction (30%)

2. Determine the reciprocal lattice of a tetragonal crystal (all angles = 90°) a = b = 5 Å, c = 8 Å Make a sketch to illustrate the unit cell in the real space and the first Brillouin zone in reciprocal space. Specify the lengths of the sides of the Brillouin zone. Label the directions in real space and reciprocal space.

3. By performing x-ray diffraction on crystallized proteins, it is possible to determine the type of crystal that is formed and the shape of the proteins. The crystal structure is not important for the biological function of the protein but the shape is very important. How is the shape of the protein determined from x-ray diffraction?

Phonons (10%)

4. Why does the phonon dispersion relation for silver look like the phonon dispersion relation for gold?

Electrons(40%)

5. Aluminum has a molar mass of 26.98 g/mol and a mass density of ρ= 2.70 g/cm³. Calculate the electron density under the assumption that each atom contributes 3 valence electrons to the free electrons. Calculate the Fermi energy and the Fermi temperature of aluminum, $k_B T_F = E_F$

6. A silicon pn junction is doped with 10^{17} cm^{-3} donors on the n-side and 10^{17} cm^{-3} acceptors on the p-side. (useful data for Silicon: band gap 1.12eV, effective density of states in conduction band (300K) $2.8 \times 10^{25} m^{-3}$, effective density of states in the valence band (300K) $9.84 \times 10^{24} m^{-3}$, Boltzmann constant $k_B = 8.6 \times 10^{-5} eV/K$)
    a. (a) Calculate the chemical potentials on the two sides at 300 K. Set the zero of energy to be at the top of the valence band.
    b. (b) Calculate the built-in voltage for this diode at 300 K.
    c. (c) Draw the electric field as a function of position indicating the direction the field is pointing.

7. Describe two experimental techniques (what are their important parts and what do they measure) that can be used to measure the electron density of states?