Introduction to Solid State Physics 2018

1. The interplay between crystalline structure and electronic structure is one of the crucial subjects in the solid state physics.
   (a) Please explain the origin of existence of the band gap from the crystalline structure by using a simple model of free electrons. (10%)
   (b) Please show how the energy gap can be determined by a certain potential height in the crystalline structure. (10%)

2. The band structure of phonons has similar but different feature as compared to that of electrons in solid states. Please point out and explain the difference between both cases in terms of momentums or $\boldsymbol{k}$ vectors in the Brillouin Zones. (15%)

3. (a) Please show that $\Delta k = G$ is the X-ray diffraction condition in a crystalline structure. (10%)
   (b) Please show that $\Delta k = G$ also indicates a condition existing at the boundaries of the Brillouin Zones of the $\boldsymbol{k}$ space. (Hint: consider the construction of Brillouin Zones through Wigner-Setz cell of the reciprocal lattice.) (10%)
   (c) However, if one considers the symmetry of the lattice, $\Delta k = G$ doesn't guarantee the diffraction pattern existing. Please derive the structure factor S_G, by using atomic form factor f_j, $\boldsymbol{G}$ vector and primitive basis for expression! Please prove that for BCC lattice , (100)、(111) or (300) pattern will not exist. (15%)

4. Please explain both conceptually and as quantitatively as possible the existence of the Landau level in a Fermi surface in the presence of a magnetic field $\boldsymbol{B}$. (Hint: The orbit of an electron, and in turn the magnetic flux contained within the orbit in real space is quantized.) (20%)

5. Please give one example of solid state systems, in which the band dispersion (at so-called Dirac Points) around the Fermi Level is linear, and also show that the band electron with linear dispersion can be considered as massless particle in analogy to the photon. (10%)