黎曼幾何與物質光學性質

量子態的幾何學是多年來理解電子系統對靜態電磁場響應的基礎。如量子霍爾效應源自於二維電子氣電子態的貝里曲率及其積分的拓撲不變性。然而，如何將量子幾何與共振光學響應作聯接則頗具挑戰性，主要困難是光躍遷涉及一對狀態，而已知幾何量是為單個狀態定義的。迄今為止，光學響應的幾何理解僅限於兩個狀態的系統，因為其希爾伯特空間完全是由單一狀態決定的。最近郭光宇教授和海外合作者通過將偶極矩躍遷矩陣元素看作復向量空間的切向量，引進黎曼度量張量、度量連接和曲率，為量子材料的任意高階光學響應構建了一套黎曼幾何理論(參見 Fig. 1 和[1])。郭教授等並應用該理論預言在一些真實材料(如 germanene, Bi_2Se_3 & $(LaOsO_3)_2$ bilayer)中三階光伏霍爾效應是由黎曼曲率張量主導的，並提出驗證此預言的實驗。

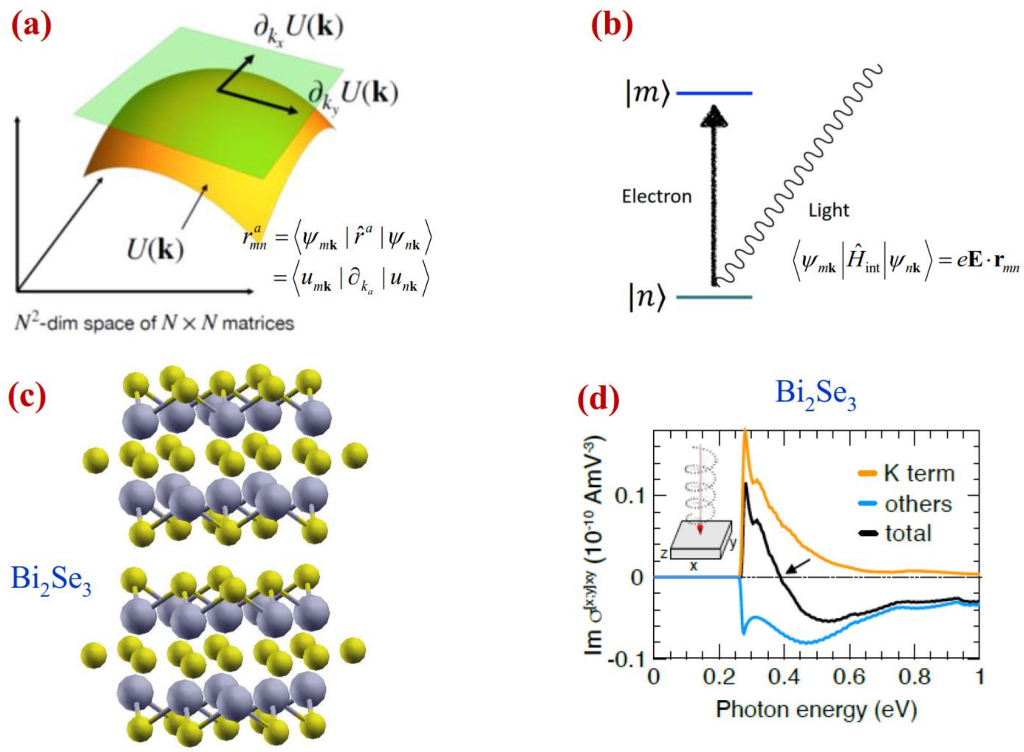

Fig. 1 (a,b) Geometry of the Bloch state (a) and optical transition (b): Transition matrix elements as tangent basis vectors. Optical transitions between *m* and *n* states reveal geometrical information on the curve generated by $\partial_{ky}U(\mathbf{k})$. **(c,d)** Calculated third-order circular photovoltaic Hall conductivity for massive Dirac material Bi_2Se_3: As theory predicts, the Hermitian curvature (K term) dominates the response near the band edge, and the Hall conductivity changes sign as the photon energy increases.

[1] J. Ahn, G.-Y. Guo, N. Nagaosa and A. Vishwanath, Riemannian Geometry of Resonant Optical Responses, Nature Physics (https://doi.org/10.1038/s41567-021-01465-z).